Minimizing Tardiness in Stochastic Flexible Job Shop Problem using a simulation based optimization approach

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Abstract. In this paper, Flexible Job Shop Scheduling Problem (FJSP) with Parallel batch processing machine is investigated. The problem is to find the best solution for assign jobs to machines and batch's processing sequence to minimizing tardiness. First a Mixed Integer Programming (MIP) formulation is proposed for the first time. Primarily proposes solution method is based on the genetic algorithm to solve the deterministic problem. The results obtained from the computational experiments validate the effectiveness of the proposed method. In addition, as the scheduling is affected by uncertain conditions, a simulation-based optimization algorithm is developed to tackle these uncertainties. This algorithm benefits from the fast-computational time and solution quality of the proposed genetic algorithm, combined with simulation technique. The uncertain factors considered include the order arrival time, Rework rate. In this approach, a simulation model is used to investigate the effect of possible conditions on the responses of the genetic algorithm.

Keywords: Batch Scheduling, Flexible job shop scheduling, Uncertainty, Simulation- based Optimization, Genetic algorithm

1 Introduction and Literature review

Scheduling can be defined as the assignment of customer orders to available resources (including personnel, machines, tools, etc.) and determining the appropriate sequence of jobs in order to optimize one or more goals [1]. Scheduling problems are divided according to the type and composition of production resources and equipment or the type of production environment. One type of scheduling problem is the job shop schedule (JSP), developed by Mann [2] more than 50 years ago. In the classical job shop scheduling problem, *n* jobs are processed on *m* machines, each job consists of *m* operations with fixed and known processing time and the processing route of each job is also known in advance. Each machine is continuously available from time zero and the operations are processed without preemption. The flexible job shop scheduling problem (FJSP) is the generalized version of the job shop scheduling problem. In this case, in addition to sequencing the jobs on the machines, the appropriate allocation of job to the machines is also done. This is also an NP-hard problem [3, 17-26] and is much more difficult than the job shop problem, for it includes the assignment of operations to machines. The flexible job shop problem is divided into two separate sub problems: (1) assigning operations to machines, and (2) sequencing operations on machines. Accordingly, various solution methods have been proposed for flexible job shop scheduling. These methods can be divided into two main categories: (1) hierarchical approaches and (2) integrated approaches. Axia et al [4] have focused on hierarchical approach to solving the flexible job shop scheduling problem. They used the particle swarm algorithm to assign operations to machines and the genetic algorithm to schedule operations.

Recently, integrated techniques have been developed using meta-heuristics methods to calculate nearly optimal solutions for deterministic FJSP. Between these methods, simulated annealing optimization algorithm by Kaplanoglu [5] and quantum behaved particle swarm optimization (QPSO) with mutation operator by Ranjan & Mahapatra [6] proposed to solve deterministic FJSP. However, in most of the real-world manufacturing environments, the uncertainty and complexity are two inherent features. Another class of scheduling problem is deterministic and uncertain problems. In deterministic type at the beginning of the schedule, all jobs and related parameters, including: processing time, job arrival time, production operation route, resource access, etc. are specified and fixed. In the probable case the disruptions may arise from new jobs arrival or job cancellations, urgent jobs to be taken into account, processing times changes, machine failures, etc. Thus, uncertainty is a very important characteristic that researchers should not deny or neglect in the problem resolution. Recently, research on production scheduling under uncertainty has attracted substantial attention [7]. Nevertheless, when incorporating the data uncertainty in the formulation of the already NP-hard FJSP, it is essential to use solution methods that prevent it from becoming more complicated. Lee Ni et al [8] proposed gene expression programming approach to solve FJSP with random arrival times of orders. Then, they performed simulation experiments to evaluate the performance of responsive programming policies developed with the GEP-based approach under different processing conditions. Kwandakchi et al [9] proposed a hybrid genetic algorithm to solve FSJP with random machine breakdowns and Random arrival time.

Other type of scheduling problem is batch scheduling. In this type of problem, machines have the ability to process batches of jobs. The batching allows multiple jobs to be simultaneously processed as long as the total size of the batch does not exceed machine capacity. The processing time of a batch is dependent on the individual jobs in the batch, which is the maximum of individual processing times. The FJSP with batch processing machine inherits every complexity of the original FJSP. In addition, it has a set of parallel batch processing machines. Each job has to be processed on one machine out of a set of given compatible machines as it visits a predetermined series of steps. In fact, the purpose of solving such problems is to find the right answer to the following two questions:

1- Which work belongs to which category?

2- What is the sequence of categories on machines?

Andy Hamm [10] considered a deterministic FJSP with batch processing machines. Hamm propose a new mathematical modeling for the problem and shown that the use of batching techniques for products leads to a reduction in completion time.

From the literature review, it is clear that batch processing can have a significant effect on reducing order completion time. However, little research has been done on flexible job shop scheduling by considering order batching. In this paper, the flexible job shop schedule with batch processing machine is considered. we propose a simulation-based optimization algorithm to solve the FJSP under uncertainty. We restrict the term of uncertainty to arrival time and rework, which refers to the repeat one or more production operations at random to attained quality requirements.

2 Problem Description

In this case, orders have a specified size and delivery date and are arrival at random times. Therefore, the order arrival time is one of the random parameters. Each order includes one type of product and each product has a specific production operation rout. Each order are placed in batches according to the time of arrival, the size of each batches should not exceed the capacity of the machines. Therefore, the processing time of each batch on the machine depends on the size of the batch and the speed of the machine to which it is assigned.

This section presents the optimization model for the deterministic FJSP. This integrated model simultaneously determines the batching of orders, the assignment of jobs to the appropriate machine, and the determination of the appropriate sequence of batches on each machine. The optimization model for the deterministic FJSP is developed based on the model proposed by Roshanaei et al [11]. Parameters and decision variables used throughout the paper are summarized as follow:

Sets

J	The set of jobs indexed by $j \in \{1, 2, \ldots, J\}$
O	The set of orders indexed by $o \in \{1, 2, \ldots, O\}$
B	The set of batches indexed by $b \in \{1, 2, \ldots, B\}$
M	The set of machines indexed by $m \in \{1, 2, \ldots, M\}$
S	The set of Processing priority by $S \in \{1, 2, \ldots, S\}$

Parameters

S_o	The size of order o	du_o	Delivery date for order o
B_o	Maximum capacity for each batch of orders o	r_o	The arrival time for order o
A_{jbm}	A binary variable set to 1 if job j from batch b can be processed on machine m and 0 otherwise	V_m	Speed of the machine m

Decision variables

X_{jbm}	1 if job *j* from batch *b* processed on machine *m* and 0 otherwise	B_b	The size of batch *b*
X_{jbms}	1 if job j from batch *b* processed on machine *m* in priority *s* on this machine and 0 otherwise	Z_b	Time to start processing operations of batch *b*
X_{ob}	1 if order *o* is inside batch *b* and 0 otherwise	CT_{bm}	Completion time of batch *b* on machine *m*
ST_{bm}	Time to start processing of operations batch *b* on machine *m*	P_{bm}	The processing time of batch *b* on machine *m*
P_{jbm}	The processing time of job *j* from batch *b* on machine *m*	ST_{jbm}	Time to start processing operations of job *j* from batch *b* on machine *m*.
CT_O	Completion time of order *o*		

2.1 Model Formulation

$$min \sum_{o} T_o \tag{1}$$

s.t.

$$T_o = max(0, CT_o - du_o) \quad ; \forall\, o \tag{2}$$

$$\sum_{m} x_{jbm=1} \quad ; \forall j\, , b \tag{3}$$

$$x_{jbm} \leq A_{jbm} \quad ; \forall j\, , b, m \tag{4}$$

$$\sum_{m}\sum_{s} x_{jbms} = 1 \;\; ; \forall j\, , b, m, s \tag{5}$$

$$\sum_{s} x_{jbms} = x_{jbm} \quad ; \forall j\, , b, m, s \tag{6}$$

$$\sum_{b} x_{ob} = n_o \;\; ; \forall o \tag{7}$$

$$x_{ob} \times B_o \leq B_b \quad ; \forall o\, , b \tag{8}$$

$$\sum_{j}\sum_{b} x_{jbms} \leq 1 \quad ; \forall m\, , s \tag{9}$$

$$CT_{jb} \geq ST_{jb} + P_{jb} \;\; ; \forall j\, , b \tag{10}$$

$$P_{jb} \geq \sum_{m} x_{jbm} \times P_{jbm} \quad ; \forall j\, , b \tag{11}$$

$$P_{jbm} = \frac{B_b}{V_m} \;\; ; \forall j\, , b, m \tag{12}$$

$$ST_{jb} + P_{jb} \leq ST_{j+1} \quad ; \forall j\, , b \tag{13}$$

$$ST_{ms} + \left(P_{jbm} \times x_{jbms}\right) \leq ST_{m(s+1)} \quad ; \forall j\, , b, m, s \tag{14}$$

$$ST_{jb} \geq r_b \quad ; \forall j\, , b \tag{15}$$

$$r_b \geq r_o - L(1 - x_{ob}) \ ; \forall o\,, b \tag{16}$$

$$\sum_b x_{ob} \geq 1 \ ; \forall o \tag{17}$$

$$\sum_j \sum_b x_{jbm(s+1)} \leq \sum_j \sum_b x_{jbms} \quad ; \forall m\,, s \tag{18}$$

$$ST_{ms} \leq ST_{jb} + L\left(1 - x_{jbms}\right) \ ; \forall j\,, b, m, s \tag{19}$$

$$ST_{ms} \geq ST_{jb} + L\left(1 + x_{jbms}\right) \ \ ; \forall j\,, b, m, s \tag{20}$$

The objective function aims to minimise total tardiness. The expression (2) shows how tardiness is calculated from the difference between the completion time and the delivery date of the order. Constraint (3) indicates that each job j can only be assigned to one machine. Since there is a set of machines for processing each job, constraint (4) ensures that each job is assigned to a machine that can process it. Constraint (5) indicates that any job j related to batch b is processed only on a specific priority s of the machine to which it is assigned. Constraint (6) ensures that if a job is not assigned to a machine, it cannot be done in any of its priorities. Constraint (7) specifies the number of batch of order o. Constraint (8) shows how to calculate the size of each batch of order o. Constraint (9) ensures that not all processing priorities of a machine can be used. In thise paper, the processing time of jobs is a function of the size of the batch and the speed of the machine to which the batch is allocated for processing. Therefore, constraints (10) - (12) are used to calculate the processing time of batched jobs. Constraint (13) ensures that prioritized relationships are maintained between the jobs of a batch. Constraint (14) ensures that there is no interference with the operation of the machines. Constraints (15) and (16) calculate the availability time of batched jobs. Constraint (17) ensures that at least one batch must be created for each order. The constraint (18) is related to the processing priorities of the machines, and indicates that if a job is assigned to a machine, the processing priorities of that machine will be reduced for subsequent assignments. Limitations (19) and (20) related to the processing time of jobs are in the processing priorities of the machines.

2.2 Simulation-based optimization approach

Simulation and optimization (SBO) methods have been used independently to solve scheduling problems. Optimization methods are able to create optimal or near-optimal solutions. Incorporating stochastic modes and the nature of system dynamics into optimization methods increases the computational complexity of these methods [12]. On the other hand, simulation techniques are able to apply system uncertainties. Because by it the internal and external factors of change in the system and the impact of these factors on performance criteria can be easily assessed. The SBO method is presented to combine the advantages of both optimization and simulation methods as a tool to solve complex problems with random factors that we are not able to solve by analytical optimization methods. SBO is an efficient decision tool for considering the relationships and interactions between the entities of a complex and random system [13]. Kulkarni et al [14] have proposed a simulation based optimization method to solve the deterministic job shop scheduling problem. In this method, a connection is made between a discrete event simulation model and a linear mathematical model. The production environment is modeled by discrete event simulation with the aim of reducing the maximum completion time. The task of the optimization model is to improve the performance of the objective function and find the optimal solution to the problem according to the feedback received from the simulation model. For this purpose, the simulation objective function is added to the mathematical model as a dynamic constraint to reduce the answer search space. Finally, by comparing the proposed approach and the methods used in the literature, the effectiveness of this method in solving large-scale problems has been proven. Sharma et al [15] consider a job shop scheduling. In thise case, jobs arrival times, processing times and sequence-dependent times have random behavior. To solve this problem, a discrete event simulation model has been used to consider the stochastic behavior of the parameters with the aim of minimizing the average tardiness and maximum completion time. They have stated that scheduling problems in large and complex production systems can only be solved using simulation methods.

In the optimization part, considering that the deterministic FJSP with batch processing machines is one of the NP-hard problems, a genetic algorithm is used to create optimal solutions to the problem. Genetic algorithms have been tested in a large number of studies and researches and the results indicate that it is one of the strongest and most popular meta-heuristic methods in the field of optimization [16].

In the simulation section, simulation is used to evaluate each response generated in each generation of the genetic algorithm under conditions of uncertainty due to random arrival times of orders and the possibility of Rework. For this purpose, the decision variables x_{jbms}, x_{jbm} and x_{ob}, which represent the allocation and sequence decision variables and were determined by the genetic algorithm in the previous step, along with the distribution function of the probability of order arrival time and the probability of reworking for each job is given as input to the simulation model. In the simulation, for a GA individual a definite number N_{SIM} of a random number is generated according to the normal distribution for the arrival time of each order. In each iteration of the simulation, the orders are placed in batchs with processing priority commensurate with their arrival time. The jobs within each batch are entered into different stages of production according to the order production operations in that batch, respectively, and are processed if the machine is idle, according to the allocation decision variable obtained from the GA algorithm. Processing time depends on the size of the batch and the speed of the machine to which the job is assigned for processing. The priority of processing jobs assigned to a machine at the same time, is commensurate with the priority of batched processing of that jobs. After the simulation iterations are completed, the mean of the objective functions obtained for a GA individual is returned to the GA algorithm as a fitness function. To achieve the optimal answer, the simulation is performed by GA algorithm with a certain number of repetitions in each step. Figure 1 shows the general structure of the proposed method and how the simulation and optimization relationship relates.

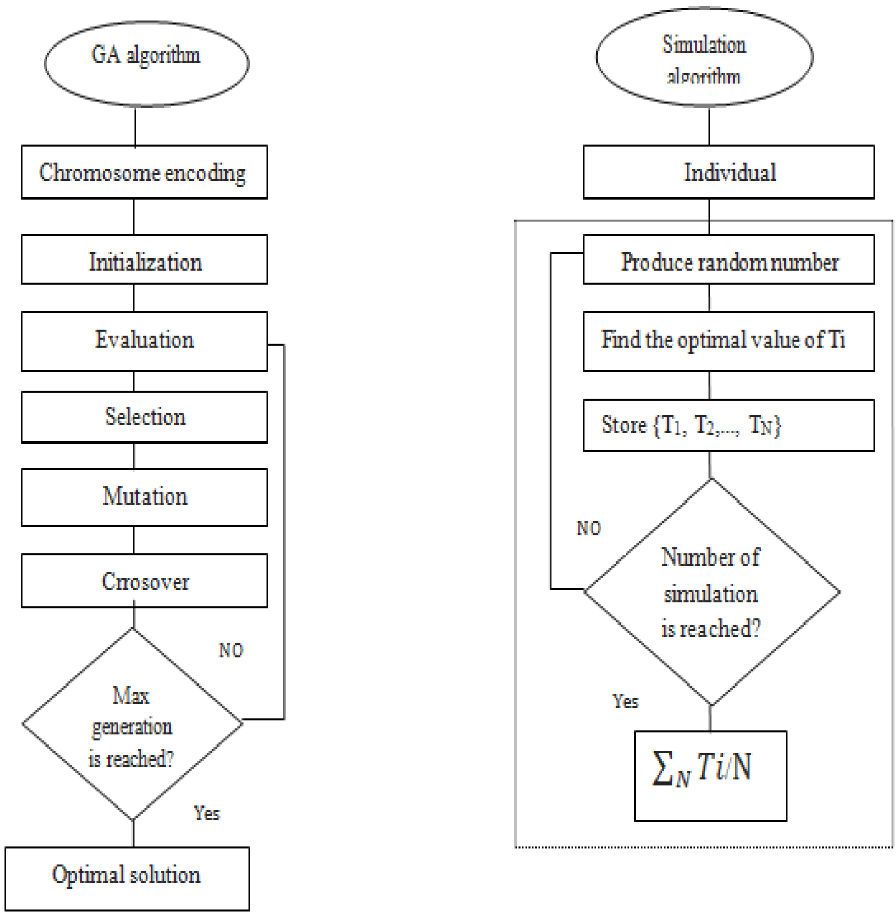

Fig 1. The framework of simulation-based optimization *proposed approach

3 Numerical Experiment

Since the genetic algorithm plays an optimizing role in the SBO method, it is very important to ensure that it produces good quality answers at the right time. To investigate and validate GA in solving problems with different dimensions, a computational experiment is created by considering different levels of problem parameters. The objective function of the solutions created by GA is then compared with the optimal solution of these problems obtained by solving the deterministic mathematical model by Gams software. The computational results are presented in Table 1. In this table *Dev* which is obtained from the (21) formulatiom, shows the percentage deviation of the value of the objective function obtained from the algorithm and Games. The existence of negative values for the *Dev* ratio indicates better performance of the algorithm in large dimensions of the problem than the exact method.

$$Dev = \frac{Algoritm - Gams}{Gams} \times 100 \qquad (21)$$

Table 1. Computational results of sample problems

					Gams			GA		
no	m	o	b	j	z	Cpu(s)	Gap	z	Cpu(s)	Dev%
1	2	2	2	2	13	2	0	13	8	0
2	2	2	4	2	15	5	0	15	10	0
3	3	3	4	2	21.5	20	0	21.5	11	0
4	3	3	3	3	43	26	0	43	12	0
5	4	4	4	3	56	2200	0	61	12	8.9
6	8	5	8	4	29.5	3600	15.41	21	21	-28.8
7	10	7	10	4	34	3600	29	11	28	-67.6
8	10	8	15	3	65	3600	64	3	33	-95.3

In the proposed stochastic optimisation model, we consider several uncertain parameters including the Order arrival time r_o and the probability of rework ret_j. In this study, we assume that these parameters follow a normal distribution where the realisations have positive outcomes. Here, we propose a simulation-based optimisation method to solve the stochastic FJSP with batch processing machines. In this method, the hybridisation of the GA and simulation is proposed. When The parameters of assigning jobs to the machines and the optimal sequence of batchs have been determined simulation is used to obtain an estimate of the total tardiness by considering the stochastic parameters. Simulation is an iterative process where at each iteration, we generate random numbers to represent the stochastic parameters. The set of data used for the stochastic FJSP is similar to the one used for the determinstic problem in Previous Section.

Table 2. Computational results of SBO approach

					SBO		GA	
No	m	o	b	j	z	Cpu(s)	z	Cpu(s)
1	2	2	4	2	18	53	12	5
2	4	4	4	3	5	200	0	12
3	8	5	8	4	7	435	0	20
4	10	7	10	4	21	600	15.5	30
5	10	8	15	3	17	640	0	33

According to the results of Table 2, it is clear that the amount of delay created in the proposed algorithm is greater than solving the deteminstic problem. However, considering the possible conditions will bring the obtained results closer to the real results of the problem Also, the use of simulation technique is able to consider the effect of possible conditions on the problem without the need to use complex analytical models.

4 Conclusions and Future Research

In this study, we investigated the batche scheduling flexible job shop problem for minimizing total tardiness whit uncertainty. We first proposed a MIP for deterministic problem and use genetic algorithm to solve it in medium and large dimensions. To deal with uncertain conditions, we propose a simulation- based optimization algorithm for solving the stochastic problem where simulation and the proposed genetic algorithm are combined. Restrict the term of uncertainty to arrival time and rework, which refers to the repeat one or more production operations at random to attained quality requirements. Simulation technique an ideal methodology for evaluating the stochastic functions in a complex problem with random factors that we are not able to solve by analytical optimization methods.

For future research, there are a number of possible extensions of the models developed in this paper. As mentioned, the mathematical model of the problem is presented in a deterministic state and the solution of the scheduling problem is done under random factors in a simulation-based optimization method. Therefore, it is suggested that uncertain conditions be added to the model and after solving with the appropriate method, the results and solution time be compared with the existing proposed method.

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