

# A Rough Set Multi-Knowledge Extraction Algorithm and Its Formal Concept Analysis

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**Abstract**—Rough set theory provides an effective method to reduce attributes and extract knowledge. This paper represents a rough set multi-knowledge extraction algorithm and its formal concept analysis. The proposed algorithm can obtain multi-reducts by using rough set in decision table. The formal concept analysis is used to obtain rules from the main values of the attributes influencing the decision making and these rules build a multi-knowledge. Experimental results show that the proposed multi-knowledge extraction algorithm is efficient.

**Keywords**-Multi-Knowledge, Rough Set, Formal Concept Analysis, Rule

## I. INTRODUCTION

Rough set theory [1], [2] is a mathematical tool for attribute reduction and has been widely applied to many fields, for instance, machine learning [3], knowledge discovery [4], [5], artificial intelligence [6], etc. The aim of attribute reduction is to remove redundant knowledge on premise of keeping the decision-making ability unchanged.

The traditional method for attribute reduction is to find a good reduct [7], [8], and a single knowledge [9] is represented by a good reduct. However, when a single knowledge classifies new objects, it may cause errors. Reducts of decision table are usually not unique in real world. So the concept of multi-knowledge [10], [11] from multi-reducts is proposed.

Formal concept analysis (FCA) [12], [13] is a powerful tool for a rule extraction. FCA can be used for conjunctive description of concepts. In FCA, extension of concept is represented as the set of all objects from this concept and intension is represented as the set of common attributes from the objects. The concept lattice [14] is the core data structure of FCA. Each node of concept lattice is a concept, which is composed of extension and intension. A concept lattice is very beneficial for obtaining the important rules from multi-knowledge. Combining rough set theory and FCA [15], [16] can get more valuable knowledge. In this paper, we present a new algorithm to extract multi-knowledge. We obtain the multi-reducts using the rough set and the important rules using the FCA. The rest of the paper is organized as follows. Section II recalls the basics of rough set and FCA,

while Section III shows the details of the multi-knowledge extraction algorithm. Experiment results and analysis are provided in Section IV and the conclusion is discussed in Section V.

## II. METHODOLOGY

The rough set theory is used to find multi-reducts and FCA is used to obtain more important rules to build multi-knowledge.

### A. Formal Concept Analysis (FCA)

In FCA, data is presented by formal concept, which is defined as follows:

A formal context  $K = (G, M, I)$ , where  $G$  is the set of all objects;  $M$  is the set of all attributes;  $(g, m) \in I$  presents that all of objects in  $g \in G$  have these attributes in  $m \in M$ .

In the formal context, two mappings  $f$  about the common attributes of  $\forall O \subseteq G$  and  $g$  about the common objects of  $\forall Q \subseteq M$  can be defined as follows:

$$\begin{aligned} f(O) &= \{m \in M \mid \forall g \in O, gIm\} \\ g(Q) &= \{g \in G \mid \forall m \in Q, gIm\} \end{aligned} \quad (1)$$

A formal concept of the formal context is a pair  $(O \subseteq G, Q \subseteq M)$ , where  $f(O) = Q, g(Q) = O$ .  $O$  is called a extension of  $(O, Q)$ , and  $Q$  is called a intension of  $(O, Q)$ . We represents a extension as  $Extent(F)$  and a intension as  $Intent(F)$  for formal concept  $F$ .

The relation between subconcept and superconcept plays a outstanding role in FCA. The relation of formal concepts  $F_1$  and  $F_2$  means if  $F_1 \leq F_2$ ,  $Extent(F_1) \subseteq Extent(F_2)$  and  $Intent(F_2) \subseteq Intent(F_1)$ .

$F_1 \prec F_2$  represents the condition is not existence, which is  $\exists F$  and  $F_1 < F < F_2$  and  $F_2$  is called upper-neighbor noted  $\omega(F_1)$  of  $F_1$ .

$G = \{1, \dots, i, \dots, n\}$  and  $O_1, O_2 \subseteq G$ . The sequence of elements in  $O_1$  and  $O_2$  are ascending according to the sequence of elements in  $G$ . This kind of sequence for  $G$  and its subsets is called a lexicographical order.

$O_1 < O_2$  in the is lexicographical order defined as follows:

$$O_1 \cap \{1, 2, \dots, i-1\} = O_2 \cap \{1, 2, \dots, i-1\} \quad (2)$$

where  $i$  represents the first different element from left to right between  $O_1$  and  $O_2$ .

The minimum extent of  $O_1$  with a lexicographical order is  $O_1 \oplus i$  as follows

$$\begin{aligned} O_1 \oplus i &= g(f((O_1 \cap \{1, 2, \dots, i-1\}) \cup \{i\})) \\ i &= \arg \max_{i \in G-O_1} (O_1 < O_1 \oplus i) \end{aligned} \quad (3)$$

The concept lattice is composed of all the concepts. A node of the concept lattice is a formal concept and describes the relationship between objects and attributes. A formal concept can be form by a Hasse [17].

### B. Multi-Knowledge Extraction Based on Rough Set

A decision table is denoted by  $T = (U, C, D, V, f)$ , where  $U$  is the universe of discourse.  $C \cap D = \emptyset$ , where  $C$  is the condition attributes and  $D$  is the decision attributes. The value of  $D$  is  $(d_1, d_2, \dots, d_{|D|})$ .  $V = \bigcup_{c \in C \cup D} V_c$  ( $V_c$  is the value set of the attribute  $c$ ).  $f$  is the total decision function such that  $f(x, c) \in V_c$  for every  $c \in C \cup D, x \in U$ .

The  $D$ -positive region on equivalence classes  $U/E$  is defined ad follows:

$$POS_E(D) = \bigcup_{X \in U/E \wedge \forall x, y \in X \Rightarrow f(x, D) = f(y, D)} X \quad (4)$$

where  $U/E = \{E_1, E_2, \dots, E_m\}$  represents the partition of the condition attributes ( $E \subseteq C$ ) for  $U$ .

If  $POS_E(D) = POS_C(D)$  and  $POS_{E-\{e\}}(D) \neq POS_C(D)$  for  $\forall e \in E$ ,  $E$  is a reduct of decision table. The set including many reducts like  $E$  is called multi-reducts  $MR$ .

In order to improve the efficiency of attribute reduction algorithm, the process of attribute reduction can be based on a simplified decision table. A simplified decision table is defined as  $S' = (U', C, D, V, f)$ , where  $U' = \{u'_1, u'_2, \dots, u'_n\}$  is the set of these first elements in the set  $U/C = \{\{u'_1\}_C, \{u'_2\}_C, \dots, \{u'_n\}_C\}$ , and if the value of decision attribute in  $\{u'_i\}_C$  is the same,  $u'_i$  belongs to  $U'_{pos}$ ; if not,  $u'_i$  belongs to  $U'_{neg}$ .

The importance of attribute is defined as follows:

$$\begin{aligned} sig_E(e) &= |U'_{E \cup \{e\}} - U'_E| \\ U'_E &= \left\{ \bigcup_{X \subseteq U'_{pos} \wedge |X/D|=1} X \right\} \cup \left\{ \bigcup_{X \subseteq U'_{neg}} X \right\} \end{aligned} \quad (5)$$

where  $X \in U'/E$ ,  $E \subseteq C$  and  $\forall e \in (C - E)$ . The greater the value of  $sig$  is, the more important the attribute is.

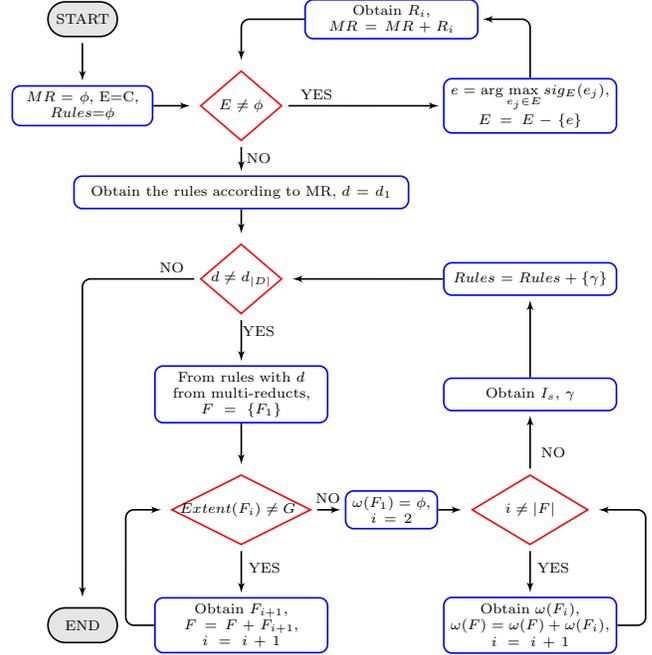


Figure 1. The flow chart of the algorithm

### III. MULTI-KNOWLEDGE EXTRACTION ALGORITHM

Firstly, a good reduct is achieved in a decision table  $T$ . Then multi-reducts can be obtained by changing the sequences of condition attributes  $C$ .  $U/C$  is got by using a quick partition based on radix sort [18]. The process for getting a reduct is repeated  $|C|$  times. Finally, the rules can be extracted by these multi-reducts.

The rules whose decision value is the same can form a context. The procedure is proposed to obtain the extensions and intensions of all the concepts of a context. Firstly, initialize a extension  $O = g(f(\emptyset))$ ,  $O \subseteq G$ .  $G$  and its subsets are sorted by the lexicographical order. The elements in  $G - O$  are selected from maximum to minimum until  $O < O \oplus l$  is the first to be satisfied. And the next extension and the next intension of  $O$  are respectively  $O \oplus l$  and  $f(O \oplus l)$ . Finally, the other extensions and intensions are calculated in accordance with the above steps until a extension is  $G$ .

The relation among concepts is obtained by upper-neighbors. Then some chief characteristics influencing the decision can be found by getting implication sets and implication relation. Finally, multi-knowledge is formed, which have many important rules from these chief characteristics. The pseudo-code for a rough set multi-knowledge extraction algorithm and its formal concept analysis (MEMFCA) is illustrated in Algorithm 1. The flow chart is illustrated in Figure 1.

#### IV. EXPERIMENTAL RESULTS OF QUALITATIVE\_BANKRUPTCY

Our algorithm was implemented in the C language. The computation environment was an Intel® Core™ i5-3337U CPU @1.80 GHz processor with 4G memory.

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#### Algorithm 1 MEMFCA Algorithm

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##### Input:

A decision table  $S = (U, C, D, V, f)$ .

##### Output:

The important rules set  $Rules$ .

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1:  $MR = \emptyset, E = C, Rules = \emptyset$ ;
2: for  $i = 1$  to  $|C|$  do
3:    $e = \arg \max_{e_j \in E} sig_E(e_j)$  using Equ. (5);
4:    $E = E - \{e\}, R_i = \{e\}$ ;
5:   while  $U' \neq \emptyset$  do
6:     Calculate  $U/R_i$  and obtain  $U', U'_{pos}, U'_{neg}$ ;
7:      $r = \arg \max_{r_t \in C-R_i} sig_{R_i}(r_t)$ ;
8:     Compute  $U'_{R_i}$  using Equ. (5);
9:      $R_i = R_i \cup \{r\}, U = U' - U'_{R_i}$ ;
10:  end while
11:  If  $R_i \notin MR, MR = MR + R_i$ ;
12: end for
13: Obtain the rules according to the multi-reducts  $MR$ ;
14: for  $d = d_1$  to  $d_{|D|}$  do
15:  Form rules with  $d$  from multi-reducts
16:  Initialize all concepts set  $F = \{F_1\}$  and  $i = 1$ , where
   $Extent(F_1) = g(f(\emptyset))$  and  $Intent(F_1) = f(\emptyset)$ ;
17:  while  $Extent(F_i) \neq G$  do
18:     $O = Extent(F_i)$ ;
19:     $l = \arg \max_{l \in G-O} (O < O \oplus l)$ ;
20:     $Extent(F_{i+1}) = O \oplus l,$ 
     $Intent(F_{i+1}) = f(O \oplus l)$ ;
21:     $F = F + F_{i+1}, i = i + 1$ ;
22:  end while
23:   $F'_i = F_{|F|-i+1}$ ;
24:   $F' = \{F'_1, F'_2, \dots, F'_{|F|}\}, \omega(F'_1) = \emptyset$ ;
25:  for  $i = 2$  to  $|F|$  do
26:    for  $j = 1$  to  $i$  do
27:      If  $F'_i \prec F'_j, \omega(F'_i) = \omega(F'_i) \cup \{j\}$ ;
28:    end for
29:  end for
30:  Obtain implication sets  $I_s$  from concept lattices of
   $F'$ ;
31:  Compute the frequency of  $\forall m \in M$  according to  $I_s$ ;
32:  Obtain the important rules  $\gamma$  according to the frequen-
  cy of  $m$ ;
33:   $Rules = Rules + \{\gamma\}$ ;
34: end for

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##### A. Results

The experiment data is Qualitative\_Bankruptcy data from UCI. Attribute information: ( $P$ =Positive,  $A$ =Average,  $N$ =Negative,  $B$ =Bankruptcy,  $NB$ =Non-Bankruptcy).

- Industrial Risk ( $Ir$ ):  $\{P, A, N\}$
- Management Risk ( $Mr$ ):  $\{P, A, N\}$
- Financial Flexibility ( $Ff$ ):  $\{P, A, N\}$
- Credibility ( $Cr$ ):  $\{P, A, N\}$
- Competitiveness ( $Co$ ):  $\{P, A, N\}$
- Operating Risk ( $Or$ ):  $\{P, A, N\}$

The condition attributes above are denoted by  $a_1, a_2, a_3, a_4, a_5, a_6$  in turn. Class  $d$  is the decision attribute which includes  $B$  and  $NB$  in the decision table.

Firstly, multi-reducts ( $MR$ ) can be obtained by using the Algorithm 1 as follows:

$$MR = \{\{a_5, a_3, a_1\}, \{a_2, a_5, a_3\}, \{a_4, a_5, a_3\}, \{a_6, a_5, a_1\}\}$$

The rules with  $NB$  can be got by  $MR$  and presented in Table I.

Table I  
RULES WITH DECISION  $NB$  FROM MULTI-REDUCTS

No.	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	d
1	A	*	N	*	A	*	NB
2	*	*	P	*	A	*	NB
3	*	P	*	*	A	*	NB
4	*	A	A	*	A	*	NB
5	*	N	A	*	A	*	NB
6	*	A	P	*	A	*	NB
7	*	N	P	*	A	*	NB
8	*	*	N	P	A	*	NB
9	*	*	P	A	A	*	NB
10	*	*	P	P	A	*	NB
11	A	*	*	*	A	A	NB
12	N	*	*	*	A	N	NB
13	P	*	*	*	A	N	NB
14	*	*	*	*	A	P	NB
15	*	*	*	*	P	*	NB
16	*	*	A	*	A	*	NB

In Table II, we present the concepts and upper-neighbors in Table I.  $a_{ij}$  expresses the  $j$ th value of the attribute  $a_i$  [19]. For example,  $a_{32}$  represents the value of the third attribute ( $FF$ ) is 2 ( $A$ ). The Hasse of Table II is shown in Figure 2.

The implication set can be achieved by using a general method [20] in FCA and is shown in Table III. The implication relation in implication set is investigated further and the frequency of the implication is given in Table IV.

We can find some chief characteristics influencing the decision  $NB$  from the frequency of implication in Table

Table II  
ALL THE CONCEPTS IN FORMAL CONTEXT OF RULES WITH  $NB$

No.	extension	instension	upper-neighbor
1	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16	$\emptyset$	$\emptyset$
2	1,2,3,4,5,6,7,8,9,10,11,12,13,14,16	$a_{52}$	1
3	1,8	$a_{12}, a_{52}$	2
4	1,11	$a_{12}, a_{52}$	2
5	1	$a_{12}, a_{33}, a_{52}$	3,4
6	2,6,7,9,10	$a_{31}, a_{52}$	2
7	3	$a_{21}, a_{52}$	2
8	4,5,16	$a_{32}, a_{52}$	2
9	4,6	$a_{22}, a_{52}$	2
10	4	$a_{22}, a_{32}, a_{52}$	8,9
11	5,7	$a_{23}, a_{52}$	2
12	5	$a_{23}, a_{32}, a_{52}$	8,11
13	6	$a_{22}, a_{31}, a_{52}$	6,9
14	7	$a_{23}, a_{31}, a_{52}$	6,11
15	8,10	$a_{41}, a_{52}$	2
16	8	$a_{33}, a_{41}, a_{52}$	15,3
17	9	$a_{31}, a_{42}, a_{52}$	6
18	10	$a_{31}, a_{41}, a_{52}$	15,6
19	11	$a_{12}, a_{52}, a_{62}$	4
20	12,13	$a_{52}, a_{63}$	2
21	12	$a_{13}, a_{52}, a_{63}$	20
22	13	$a_{11}, a_{52}, a_{63}$	20
23	14	$a_{52}, a_{61}$	2
24	15	$a_{51}$	1
25	$\emptyset$	$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{41}, a_{51}, a_{52}, a_{61}, a_{62}, a_{63}$	5,7,10,12,13,14,16,17,18,19,21,22,23,24

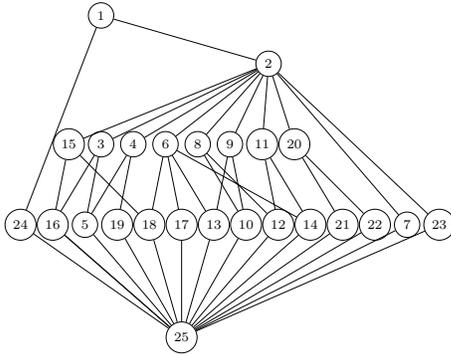


Figure 2. The Hasse of Concept Lattice for Context of Rules with  $NB$

Table III  
IMPLICATION SET FOR CONTEXT OF RULES WITH  $NB$

Implication Sets		
[1] $a_{33} \Rightarrow a_{52}$	[6] $a_{22} \Rightarrow a_{52}$	[11] $a_{63} \Rightarrow a_{52}$
[2] $a_{12} \Rightarrow a_{52}$	[7] $a_{23} \Rightarrow a_{52}$	[12] $a_{13} \Rightarrow a_{52}, a_{63}$
[3] $a_{31} \Rightarrow a_{52}$	[8] $a_{41} \Rightarrow a_{52}$	[13] $a_{11} \Rightarrow a_{52}, a_{63}$
[4] $a_{21} \Rightarrow a_{52}$	[9] $a_{42} \Rightarrow a_{31}, a_{52}$	[14] $a_{61} \Rightarrow a_{52}$
[5] $a_{32} \Rightarrow a_{52}$	[10] $a_{62} \Rightarrow a_{12}, a_{52}$	

IV. The most important characteristic is  $a_{52}$  which means the value of condition attribute  $Co$  is  $A$ . The next important characteristic is that the value of  $Or$  is  $N$ .

Table IV  
IMPLICATION RELATION FOR CONTEXT OF RULES WITH  $NB$

Superconcept	Subconcept	Frequency
$a_{12}$	$a_{62}$	1
$a_{31}$	$a_{42}$	1
$a_{52}$	$a_{33}; a_{12}; a_{31}; a_{21}; a_{32}; a_{22}; a_{23}; a_{41}; a_{42}; a_{62}; a_{63}; a_{13}; a_{11}; a_{61}$	14
$a_{63}$	$a_{13}; a_{11}$	2

### B. Discussion

The multi-knowledge is composed of some rules extracted according to Table I with IV. The frequency of  $a_{52}$  and  $a_{63}$  is more than a threshold which is set 1 in Table IV.

These rules including both  $a_{52}$  and  $a_{63}$  may be important and these important rules are extracted as follows:

- 1) If  $a_1 = A, a_3 = N, a_5 = A$ , then  $d = NB$ ;
- 2) If  $a_3 = P, a_5 = A$ , then  $d = NB$ ;
- 3) If  $a_2 = A, a_3 = P, a_5 = A$ , then  $d = NB$ ;
- 4) If  $a_1 = N, a_3 = P, a_5 = A$ , then  $d = NB$ ;
- 5) If  $a_3 = P, a_4 = A, a_5 = A$ , then  $d = NB$ ;
- 6) If  $a_3 = P, a_4 = P, a_5 = A$ , then  $d = NB$ ;
- 7) If  $a_1 = N, a_5 = A, a_6 = N$ , then  $d = NB$ ;
- 8) If  $a_1 = P, a_5 = A, a_6 = N$ , then  $d = NB$ ;

For example, Rule 1) represents if  $Ir$  is 'Average',  $Ff$  is 'Negative' and the  $Co$  is 'Average', then the decision is 'Non-Bankruptcy'.

These rules including either  $a_{52}$  or  $a_{63}$  may be ordinary and they are obtained as follows:

- 1) If  $a_2 = P$ ,  $a_5 = A$ , then  $d = NB$ ;
- 2) If  $a_2 = A$ ,  $a_3 = A$ ,  $a_5 = A$ , then  $d = NB$ ;
- 3) If  $a_2 = N$ ,  $a_3 = N$ ,  $a_5 = A$ , then  $d = NB$ ;
- 4) If  $a_3 = N$ ,  $a_4 = P$ ,  $a_5 = A$ , then  $d = NB$ ;
- 5) If  $a_1 = A$ ,  $a_5 = A$ ,  $a_6 = A$ , then  $d = NB$ ;
- 6) If  $a_5 = A$ ,  $a_6 = P$ , then  $d = NB$ ;
- 7) If  $a_3 = A$ ,  $a_5 = A$ , then  $d = NB$ ;

The multi-knowledge using MEMFCA algorithm is formed eight important rules from Qualitative-Bankruptcy data and seven ordinary rules.

## V. CONCLUSIONS

In this paper, the aim is to extract multi-reducts for building a multi-knowledge. The multi-knowledge is composed of more rules in order to adapt to changing conditions. The two theories, rough set and FCA, are respectively used to extract multi-reducts and obtain rules. These rules can form a multi-knowledge. The multi-knowledge using FCA is based on the chief characteristics influencing the decision for reducing complexity of decision. The experiment results represent multi-knowledge with more rules and the number of rules can be according to realistic environments.

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