Hybrid Differential Evolution – Particle Swarm Optimization Algorithm for Solving Global Optimization Problems

[1]Millie Pant, [1]Radha Thangaraj, [2]Crina Grosan and [3]Ajith Abraham
[1]*Department. of Paper Technology, IIT Roorkee, India*
[2]*Babes – Bolyai University, Cluj-Napaco, Romania*
[3]*Center of Excellence for Quantifiable Quality of Service,*
Norwegian University of Science and Technology, Norway
millifpt@iitr.ernet.in, t.radha@ieee.org, cgrosan@cs.ubbcluj.ro, ajith.abraham@ieee.org

Abstract

This paper presents a simple, hybrid two phase global optimization algorithm called DE-PSO for solving global optimization problems. DE-PSO consists of alternating phases of Differential Evolution (DE) and Particle Swarm Optimization (PSO). The algorithm is designed so as to preserve the strengths of both the algorithms. Empirical results show that the proposed DE-PSO is quite competent for solving the considered test functions as well as real life problems.

1. Introduction

In the past few years, DE and PSO have emerged as powerful optimization tools for solving complex optimization problems. Both are population based stochastic search techniques inspired by nature. Although, DE and PSO have been successfully applied to a wide range of problems including test and real life problems both have certain shortcomings associated with them. The major problems being lack of diversity resulting in a suboptimal solution [1] or a slow convergence rate. In order to improve the performance of these algorithms a number of variations have been suggested in literature, one of them being the hybridization of the two algorithms. Some hybrid versions of DE and PSO include Hendtlass approach [2], where the population evolved by DE is optimized by using PSO, Kannan approach [3]; in which DE is applied to each particle for a finite number of iterations to determine the best particle which is then included into the population. Methods of Zhang and Xie [4] and Talbi and Batauche [5] apply DE to the best particle obtained by PSO. Omran et al [6] developed a hybrid version consisting of Barebones PSO and DE. In Zhi Feng et al [7] hybrid version, the candidate solution is generated either by DE or by PSO according to some fixed probability distribution.

In this paper we propose a simple hybrid version of DE and PSO, called DE-PSO. DE-PSO starts with the usual DE and incorporated PSO to reach to the optimal solution.

The global optimization problems in this paper follow the form:

$$\text{Min } f(X) \text{ subject to } X \in \Lambda$$

where X is a continuous variable vector with domain $\Lambda \in R^n$. The domain Λ is defined within upper and lower limits of each dimension. We notate the global optimal solution by X^*, with its corresponding global optimal function value $f(X^*)$.

The remaining of the paper is organized as follows: in Sections 2 and 3, we give a brief description of DE and PSO algorithms respectively. Section 4, describes the proposed DE-PSO algorithm. Benchmark problems, some real world problems and corresponding numerical results are given in Section 5. The paper finally concludes with Section 6.

2. Differential Evolution

Differential evolution (DE) is an Evolutionary Algorithm (EA) proposed by Storn and Price in 1995 [8]. DE is similar to other EAs particularly Genetic Algorithms (GA) [9] in the sense that it uses the same evolutionary operators like selection recombination and mutation like that of GA. However the significant difference is that DE uses *distance* and *direction* information from the current population to guide the search process. The performance of DE depends on the manipulation of *target vector* and *difference vector* in order to obtain a *trial vector.* Mutation is the main operator in DE. A brief working may be described as: For a D-dimensional search space, each target vector $x_{i,g}$, a mutant vector is generated by

$$v_{i,g+1} = x_{r_1,g} + F*(x_{r_2,g} - x_{r_3,g}) \quad (1)$$

where $r_1, r_2, r_3 \in \{1,2,...., NP\}$ are randomly chosen integers, must be different from each other and also different from the running index i. F (>0) is a scaling factor which controls the amplification of the differential evolution $(x_{r_2,g} - x_{r_3,g})$. In order to increase the diversity of the perturbed parameter vectors, crossover is introduced [9]. The parent vector is mixed with the mutated vector to produce a trial vector $u_{ji,g+1}$,

$$u_{ji,g+1} = \begin{cases} v_{ji,g+1} & if\ (rand_j \le CR)\ \ or\ \ (j = j_{rand}) \\ x_{ji,g} & if\ (rand_j > CR)\ \ and\ \ (j \ne j_{rand}) \end{cases}$$

where j = 1, 2,……, D; $rand_j \in [0,1]$; CR is the crossover constant takes values in the range [0, 1] and $j_{rand} \in (1,2,....., D)$ is the randomly chosen index.

Selection is the step to choose the vector between the target vector and the trial vector with the aim of creating an individual for the next generation.

3. Particle Swarm Optimization

PSO was proposed in 1995 by Kennedy and Eberhart [10]. The mechanism of PSO is inspired from the complex social behavior shown by the natural species. For a D-dimensional search space the position of the ith particle is represented as $X_i = (x_{i1},x_{i2},..x_{iD})$. Each particle maintains a memory of its previous best position $P_i = (p_{i1}, p_{i2}… p_{iD})$ and a velocity $V_i = (v_{i1}, v_{i2},…v_{iD})$ along each dimension . At each iteration, the P vector of the particle with best fitness in the local neighborhood, designated g, and the P vector of the current particle are combined to adjust the velocity along each dimension and a new position of the particle is determined using that velocity. The two basic equations which govern the working of PSO are that of velocity vector and position vector are given by:

$$v_{id} = \omega v_{id} + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (p_{gd} - x_{id}) \quad (2)$$

$$x_{id} = x_{id} + v_{id} \quad (3)$$

The first part of equation (2) represents the inertia of the previous velocity, the second part is tells us about the personal thinking of the particle and the third part represents the cooperation among particles and is therefore named as the social component. Acceleration constants c1, c2 and inertia weight ω are predefined by the user and r1, r2 are the uniformly generated random numbers in the range of [0, 1].

4. Proposed DE-PSO

The proposed DE-PSO as mentioned earlier is a hybrid version of DE and PSO. DE-PSO starts like the usual DE algorithm up to the point where the trial vector is generated. If the trial vector satisfies the conditions given by equation (4), then it is included in the population otherwise the algorithm enters the PSO phase and generates a new candidate solution. The method is repeated iteratively till the optimum value is reached. The inclusion of PSO phase creates a perturbation in the population, which in turn helps in maintaining diversity of the population and producing a good optimal solution.
The pseudo code of the Hybrid DE and PSO (DE-PSO) Algorithm is:

Initialize the population
For i = 1 to N (Population size) do
Select $r_1, r_2, r_3 \in$ N randomly
// r_1, r_2, r_3 are selected such that $r_1 \ne r_2 \ne r_3$//
For j = 1 to D (dimension) do
    Select $j_{rand} \in D$
    If (rand () < CR or j = j_{rand})
// rand () denotes a uniformly distributed random number between 0 and 1//
    $U_{ji,g+1} = x_{r_1,g} + F*(x_{r_2,g} - x_{r_3,g})$
    End if
    If ($f(U_{ji,g+1}) < f(X_{ji,g})$) then
    $X_{ji,g+1} = U_{ji,g+1}$
    Else
    PSO activated
Find a new particle using equations (2) and (3).
    (Let this particle be TX_{ji})
    If ($f(TX_{ji}) < f(X_{ji,g})$) then $X_{ji,g+1} = TX_{ji}$
    Else $X_{ji,g+1} = X_{ji,g}$
    End if
    End if
End for
End for.

5. Experimental Settings

Experimental settings for proposed DE-PSO, DE and PSO (Table 2):
For dimension 30: Pop=30, run=30, Max Gne=3000
For dimension 50: Pop=50, run=20, Max Gne=5000
For dimension 100: Pop=100, run=10, Max Gne=10000
Experimental settings for proposed DE-PSO and DEPSO [7] (Table 4):
Pop: 30, dim: 30, Max Gne.: 12000
Experimental settings for proposed DE-PSO and BBDE [6] (Table 4):
Pop: 30, dim: 30, Max number of function evaluations: 100000

In the above mentioned settings, Pop denotes the population size taken; run denotes the number of times an algorithm is executed; Max Gne denotes the maximum number of generations allowed for each algorithm
Computer Settings:

All the three algorithms were implemented using Turbo C++ on a PC compatible with Pentium IV, a 3.2 GHz processor and 2 GB of RAM.

5.1 Benchmark Problems and Results

For the present study, a set of twelve unconstrained benchmark problems is taken (Table 1). Although this collection may not be called exhaustive but it is a good launch pad to decide the authenticity of an optimization algorithm. All the problems are scalable and are tested for dimensions 30, 50, 100. Functions f1, f3, f5-f9 are highly modal where the complexity of the problem increases with the increase in the number of variables. We have also taken a noisy function (f9), where a uniformly distributed random noise is added to the function. f11 is a discontinuous function and for f12, the optimum lies in a plateau like region.
Numerical results in Table 2 show the performance of DE-PSO with the classical PSO and DE. In Table 4, we give a comparison of DE-PSO with two other recently proposed hybrid version of DE and PSO, namely BBDE [6] and DEPSO [7].
Table 2 shows that the proposed DE-PSO algorithm gives a superior performance in comparison to both DE and PSO in almost all the test cases. Table 3 gives the number of times PSO phase is activated so as to reach the optimal solution. In [6] and [7] problems of dimension 30 are taken. For the purpose of comparison we took the problems common in [6] and [7] and followed the same experimental settings. Numerical results in Table 4 show that in comparison to [7], DE-PSO gave a better performance in four out of five test cases tied and in comparison to BBDE, DE-PSO gave a superior performance in three out of five test cases tried. For f12 both BBDE and DE-PSO gave same performance.

5.2 Real life Problems and Results

The credibility of an optimization algorithm also depends on its ability to solve real life problems. In this paper we took three real life problems to validate the performance of the proposed DE-PSO.
Gas transmission compressor design [11]:

Min $f(x) = 8.61\times10^5 \times {x_1}^{1/2} x_2 {x_3}^{-2/3} ({x_2}^2 - 1)^{-1/2}$
$+ 3.69\times10^4 \times x_3 + 7.72\times10^8 \times {x_1}^{-1} {x_2}^{0.219}$
$- 765.43\times10^6 \times {x_1}^{-1}$

Subject to: $10 \le x_1 \le 55$, $1.1 \le x_2 \le 2$, $10 \le x_3 \le 40$

Optimal thermohydralic performance of an artificially roughened air heater [12]:

Max $L = 2.51 * \ln e^+ + 5.5 - 0.1 R_M - G_H$

Where $R_M = 0.95 {x_2}^{0.53}$; $GH = 4.5(e^+)^{0.28}(0.7)^{0.57}$;

$e^+ = x_1 x_3 (\bar{f}/2)^{1/2}$; $\bar{f} = (f_s + f_r)/2$; $f_s = 0.079 {x_3}^{-0.25}$;

$f_r = 2(0.95 {x_3}^{0.53} + 2.5 * \ln(1/2x_1)^2 - 3.75)^{-2}$;

Subject to: $0.02 \le x_1 \le 0.8$, $10 \le x_2 \le 40$,
$3000 \le x_3 \le 20000$

Optimal capacity of gas production facilities [11]:

Min $f(x) = 61.8 + 5.72 x_1 + 0.2623[(40 - x_1)\ln(\frac{x_2}{200})]^{-0.85}$
$+ 0.087(40 - x_1)\ln(\frac{x_2}{200}) + 700.23 {x_2}^{-0.75}$

Subject to: $x_1 \ge 17.5$, $x_2 \ge 200$; $17.5 \le x_1 \le 40$,
$300 \le x_2 \le 600$.

Numerical results for the real life problems are listed in Table 5. Numerical results show that in terms of average number of generations required to reach the optimum solution, the proposed DE-PSO gave the best results. However in terms of function value all the algorithms gave more or less similar results.

6. Conclusions

A hybrid of DE and PSO called DE-PSO is proposed and its performnce is validated on a set of benchmark and real life problems. Numerical results show that DE-PSO outperformed the classical DE and PSO and also two recently proposed hybrid version of DE and PSO. Future research will investigate the performance of proposed DE-PSO on constrained optimization problems.

Table 1. Numerical Benchmark Problems

Function	Range	Optimum
$f_1(x) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$	[-5.12,5.12]	0
$f_2(x) = \sum_{i=1}^{n} x_i^2$	[-5.12,5.12]	0
$f_3(x) = \frac{1}{4000}\sum_{i=0}^{n-1} x_i^2 - \sum_{i=0}^{n-1}\cos(\frac{x_i}{\sqrt{i+1}}) + 1$	[-600,600]	0
$f_4(x) = \sum_{i=0}^{n-1} 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2$	[-30,30]	0
$f_5(x) = -\sum_{i=1}^{n} x_i \sin(\sqrt{\mid x_i \mid})$	[-500,500]	-418.9829*n
$f_6(x) = (0.1)\{\sin^2(3\pi x_1) + \sum_{i=1}^{n-1}((x_i - 1)^2(1 + \sin^2(3\pi x_{i+1}))) + (x_n - 1)(1 + \sin^2(2\pi x_n))\} + \sum_{i=0}^{n-1} u(x_i, 5, 100, 4)$	[-50,50]	-1.1428
$f_7(x) = \frac{\pi}{n}\{10\sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(y_{i+1}\pi)] + (y_n - 1)^2\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	[-50,50]	0
$f_8(x) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))$	[-32,32]	0
$f_9(x) = (\sum_{i=0}^{n-1}(i+1)x_i^4) + rand[0,1]$	[-1.28,1.28]	0
$f_{10}(x) = \max \mid x_i \mid, \quad 0 \le i < n$	[-100,100]	0
$f_{11}(x) = \sum_{i=0}^{n-1} \mid x_i \mid + \prod_{i=0}^{n-1} \mid x_i \mid$	[-10,10]	0
$f_{12}(x) = \sum_{i=0}^{n-1} \lfloor x_i + 1/2 \rfloor^2$	[-100,100]	0

Table 3. Number of times PSO phase is activated

f	Dim	N_{PSO}	f	Dim	N_{PSO}	f	Dim	N_{PSO}	f	Dim	N_{PSO}
f1	30	157	*f7*	30	142	*f4*	30	180	*f10*	30	68
	50	146		50	143		50	180		50	137
	100	309		100	----		100	337		100	----
f2	30	189	*f8*	30	135	*f5*	30	117	*f11*	30	174
	50	162		50	147		50	117		50	176
	100	194		100	172		100	196		100	322
f3	30	178	*f9*	30	35	*f6*	30	142	*f12*	30	63
	50	177		50	52		50	163		50	70
	100	378		100	147		100	196		100	63

Table 2. Comparison results of PSO, DE and DE-PSO for functions f1 – f12

f	Dim	PSO	DE	DE-PSO	f	Dim	PSO	DE	DE-PSO
f1	30	37.819 (7.456)	2.531 (5.19026)	**1.6141 (3.885)**	*f7*	30	0.020733 (0.0528)	5.505e-13 (0.00000)	**5.504e-13 (0.000)**
	50	75.309 (19.559)	41.470 (8.805)	**24.5788** (14.6261)		50	1.75361 (2.4126)	3.334e-13 (1.488e-15)	**3.303e-13 (0.000)**
	100	**186.045 (4.939)**	261.198 (1.64699)	251.491 (8.26771)		100	----	----	----
f2	30	3.542e-16 (4.26e-16)	2.551e-47 (3.032e-47)	**4.077e-48 (1.593e-47)**	*f8*	30	1.026e-08 (1.90e-08)	7.250e-15 (7.742e-16)	**3.697e-15 0.000)**
	50	0.004 (0.003)	6.5094e-48 (4.205e-48)	**2.753e-49 (1.401e-49)**		50	0.797 (0.423)	1.717e-13 (3.865e-14)	**7.250e-15 (1.565e-15)**
	100	0.070 (1.43e-03)	2.117e-39 (3.576e-39)	**1.093e-40 (1.769e-40)**		100	2.53057 (1.45432)	1.435e-14 (1.790e-14)	**1.025e-15 (2.501e-14)**
f3	30	0.0184 (0.023)	**0.00000 (0.00000)**	**0.000 (0.000)**	*f9*	30	0.508 (0.2508)	**0.0074 (0.001)**	0.0076 (.002)
	50	0.381 (0.173)	**5.421e-20 (0.00000)**	**5.421e-20 (0.00000)**		50	0.147 (0.035)	0.012 (0.002)	**0.011 (0.001)**
	100	1.051 (0.618)	8.131e-19 (1.154e-17)	**1.084e-19 (2.493-18)**		100	0.781 (0.052)	0.0531 (0.021)	**0.0358 (0.015)**
f4	30	81.273 (41.218)	31.1369 (17.1211)	**24.202 (12.3086)**	*f10*	30	5.357 (3.204)	4.239e-06 **(1.32e-06)**	**2.474e-06** (1.440e-06)
	50	174.222 (113.635)	50.3377 (16.8557)	**44.741 (1.402)**		50	17.643 (2.027)	0.0005 (0.0004)	**3.697e-05 (7.255e-06)**
	100	250.681 (24.643)	91.2370 (3.82465)	**91.024 (3.400)**		100	----	----	----
f5	30	-10652.33 (663.174)	-12534 (54.2753)	**-12545.8 (47.3753)**	*f11*	30	2.063e-11 (5.853e-12)	8.915e-27 **(3.348e-27)**	**4.792e-27** (4.523e-27)
	50	-16685.7 (372.981)	-20818.9 (123.653)	**-20913.6 (54.2753)**		50	0.0681 (0.068)	1.670e-26 (5.592e-27)	**2.320e-27 (1.216e-27)**
	100	-30417.7 (530.050)	**41898.3 (0.000)**	**41898.3 (0.000)**		100	1.7619 (0.976)	1.779e-22 (9.021e-26)	**1.016e-22 (1.191e-23)**
f6	30	-1.1384 (0.0052)	-1.149356 (1.857e-16)	**-1.150 (0.000)**	*f12*	30	0.05 (0.217)	**0.000 (0.000)**	**0.00000 (0.00000)**
	50	37.0296 (22.9197)	**-1.15044 (0.00000)**	**-1.15044 (0.000)**		50	3.1 (2.507)	**0.000 (0.000)**	**0.00000 (0.00000)**
	100	173.854 (11.330)	**-1.150 (0.000)**	**-1.150 (0.000)**		100	63.5 (4.5)	**0.000 (0.000)**	**0.00000 (0.00000)**

Table 4. Comparison of DE-PSO with DEPSO [7] and BBDE [6]

f	DEPSO [7] Mean (Std)	DE-PSO (This paper) Mean (Std)	f	BBDE [6] Mean (Std Dev)	DE-PSO (This paper) Mean (Std Dev)
F1	24.216 (6.417)	**1.6141(3.885)**	F1	72.185 (3.018)	**1.6141 (3.885)**
F3	6.2e-16 (4.1e-16))	**0.000 (0.000)**	F3	0.269e-01 (0.767-02)	**0.000 (0.000)**
F5	-12547.7 (66.25)	**-12554.3** (95.304)	F4	**14.295** (0.948)	24.202(12.3086)
F7	**3.9e-20** (4.1e-21)	5.505e-013(0.000)	F8	2.1361 (0.159)	**3.697e-015 (0.000)**
F8	-0.0002 (0.0002)	**3.697e-015 (0.000)**	F12	0.000 (0.000)	**0.000 (0.000)**

Table 5. Numerical results of Real Life Problems

Gas Transmission Compressor Design				
Item	PSO	DE	DE-PSO	[11]
x_1	55	51.9857	53.4474	55
x_2	1.19541	1.18335	1.1901	1.195
x_3	24.7749	24.7195	24.7186	25.026
f(x)	296.446 e+004	296.448 e+004	**296.436 e+004**	296.455 e+004
G_{Avg}	786.7	146.4	**129.6**	NA
f_{Eval}	23631	4422	6205.1	NA
N_{PSO}	NA	NA	34	NA
Optimal Thermohydralic Performance of an Artificially Roughened Air Heater				
Item	PSO	DE	DE-PSO	[12]
x_1	0.05809	0.12469	0.15301	0.052
x_2	10	10	10	10
x_3	10400.2	3811.07	3000	10258
f(x)	4.21422	4.21422	4.21422	4.182
G_{Avg}	205.9	87.4	**83.9**	NA
f_{Eval}	6207	2652	4115	NA
N_{PSO}	NA	NA	16	NA
Optimal Capacity of Gas Production Facilities				
Item	PSO	DE	DE-PSO	[11]
x_1	17.5	17.5	17.5	17.5
x_2	600	600	600	465
f(x)	169.844	169.844	169.844	173.76
G_{Avg}	10.4	15.1	**9.9**	NA
f_{Eval}	342	483	423	NA
N_{PSO}	NA	NA	44	NA

G_{avg} – Average number of generations
N_{feval} – number of function evaluations
N_{PSO} – Number of times PSO activated

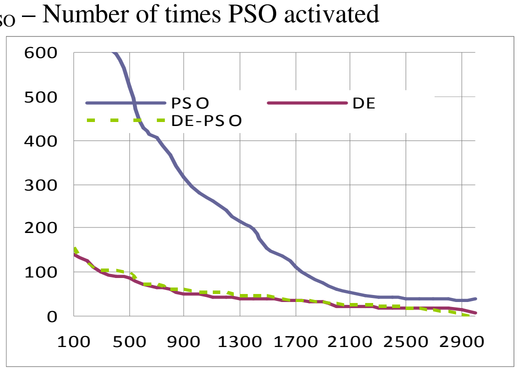

Figure 1(a)

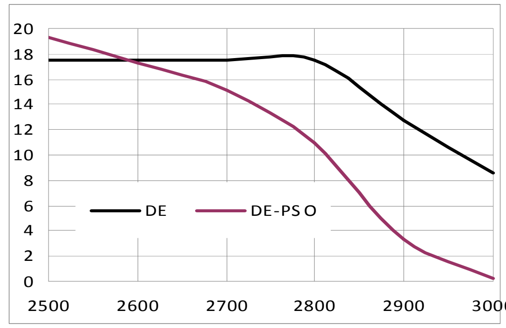

Figure 1(b)

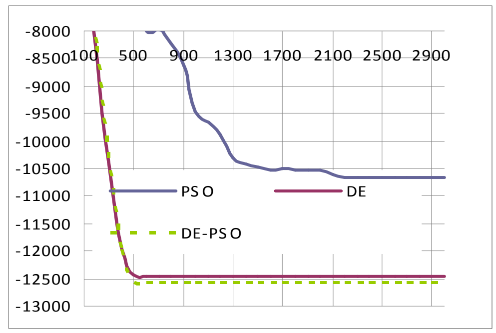

Figure 2(a)

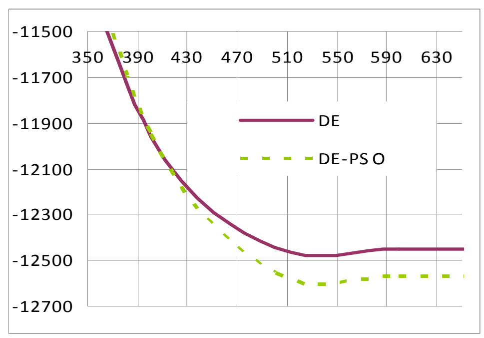

Figure 2 (b)

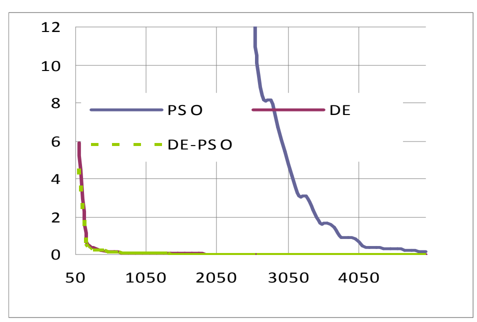

Figure 3 (a)

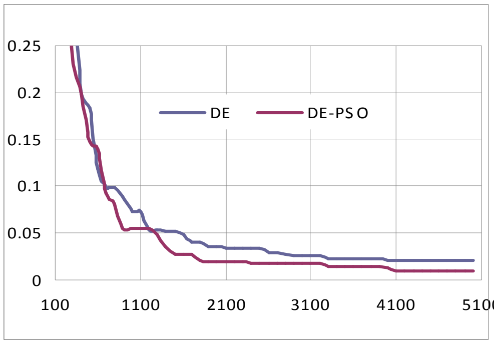

Figure 3 (b)

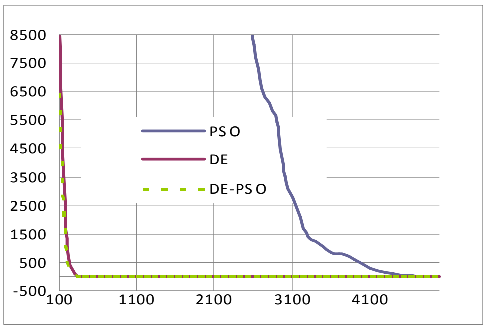

Figure 4(a)

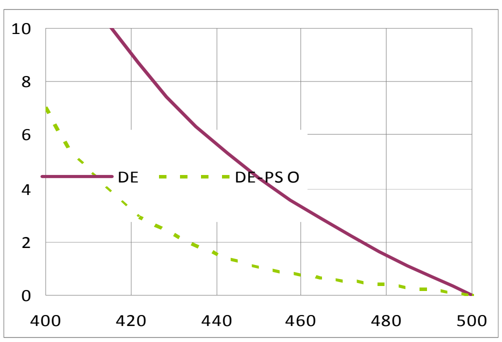

Figure 4 (b)

Figures 1(a) – 4(a) : Performance comparison of DE-PSO with DE and PSO for functions f1, f5, f9 and f12
Figures 1(b) – 4(b) : Performance comparison of DE-PSO with DE f1, f5, f9 and f12 for the last 1000 generations. In all the above figures, the horizontal axis represents the generation and the vertical axis represents the fitness function value

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