

Feedforward and Feedback Optimal Vibration Rejection for Active Suspension Discrete-Time Systems under In-Vehicle Networks

Shi-Yuan Han, Yue-Hui Chen*, Kun Ma, Dong Wang
Shandong Provincial Key Laboratory of Network
Based Intelligent Computing, University of Jinan
Jinan, 250022, China

Email: {ise_hansy, yhchen, ise_mak, ise_wangd}@ujn.edu.cn

* Corresponding author: yhchen@ujn.edu.cn.

Ajith Abraham^{1,2}

¹ IT4Innovations, VSB-Technical University of Ostrava,
Ostrava, Czech Republic

² Machine Intelligence Research Labs (MIR Labs),
Auburn, Washington, USA
Email: ajith.abraham@ieee.org

Zhong-Guang Liu

Vehicle Management Agency of Jinan Public Security Bureau,
Jinan, China, 250000

Email: liuzgcn@163.com

Abstract—This paper studies the vibration rejection problem of active suspension discrete-time systems under in-vehicle networks and designs a controller of feedforward and feedback optimal vibration rejection. Based on the ground displacement power spectral density, an discrete-time exosystem is employed to estimate the random road disturbances. A two degree of freedom discrete-time system is introduced to describe the active suspension under in-vehicle networks. Then, the original vibration control is formulated as the optimal control for a linear discrete-time system affected by external disturbances. The feedforward and feedback optimal vibration rejection law (FFOVL) is designed by solving the Riccati and Stein equations, in which the feedforward term incorporates the information of the random road disturbances and the feedback loop includes the status of suspension system. The feasibility and effectiveness of the proposed approaches are validated by an active suspension structure.

Keywords—active suspension discrete-time systems; in-vehicle networks; optimal vibration rejection; feedforward control; feedback control;

I. INTRODUCTION

Vehicle suspension plays an important role in vehicle design, which goes through three states: passive [1], [2], semi-active [3], and active suspension systems [4]–[6], in the last few decades. It is well known that active suspension systems can provide the optimization performances to ensure the driving safety, vehicle handling, and ride comfort by using power sources, such as compressors and hydraulic pumps. Active suspension system can turn into practical ones by using low consumption elements. What's more, the vibration of the vehicle body could be diminish effectively in active suspension systems, where the suspension characteristics can be adjusted by using power sources while driving to accommodate the profile of the road [7]. With the low consumption and preferable vibration, the theoretical and experimental research effort of active suspension has been considered in the last few decades,

such as H_∞ control [6], [8], sliding mode theory [9], adaptive control [10], and fuzzy control [11].

With the development of communication networks in vehicle, in-vehicle networks have been widely used in vehicle systems [12], such as Controller Area Network (CAN), Local Interconnect Network, Media Oriented Systems Transport, and so on. Actually, in-vehicle networks could be viewed as a high-speed networks, in which the information, driving state and road roughness, could be provided and communicated precisely. Then, the information provision for vehicle's behaviours is guaranteed. Therefore, it is necessary to design the model and research the control algorithm for active suspension vehicle under in-vehicle networks.

In practical application, there are three critical elements in any active vehicle suspension design and control: ride comfort, tire deflection, and suspension deflection. As is well known, ride comfort is mainly related to the road roughness disturbances. Vehicle handling and driving safety is mainly depended on the uninterrupted contact of wheels. At the same time, suspension deflection may result in deterioration of ride comfort and even structural damage. In order to ensure the performance index within an acceptable level, the road roughness disturbances must be considered in the design of active suspension. In order to reduce the influences caused by the road roughness disturbances in vehicle suspension system, the vibration rejection problem for vehicle active suspension system could be formulated to ensure the ride comfort, tire deflection, and suspension deflection at an acceptable level [8], [12]. It should be noted that the irregular road disturbances could be estimated as the random process with a ground displacement power spectral density [13].

This paper investigates the feedforward and feedback optimal vibration rejection for an active vehicle suspension discrete-time system under in-vehicle networks. The proposed

FFOVRL is developed to reduce the random road disturbances. First, the random road disturbances are viewed as an exosystem based on the characters of road disturbances. The active suspension under in-vehicle networks is simplified into a discrete-time system under external disturbances. Then, the original vibration control is formulated as the optimal vibration control for discrete-time active suspension system under in-vehicle networks. The FFOVRL is obtained by using the minimum principle, in which the status of suspension system construct the feedback term, and the information of the random road disturbances incorporates the feedforward term. A numerical example of the FFOVRL law for an active vehicle suspension with under road disturbances is presented to demonstrate the effectiveness of the FFOVRL law.

The rest of paper is organized as follows. Section II presents an introduced exosystem to describe the random irregular road disturbances. The model of active suspension discrete-time systems under in-vehicle networks and quadratic performance index are introduced in Section III. Section IV presents the FFOVRL based on the minimum theory. Numerical examples are given in Section V to demonstrate the effectiveness of the FFOVRL. The last section gives the conclusion of the paper.

II. IRREGULAR ROAD ROUGHNESS DISTURBANCES

The structures of vehicle suspension are exposed to sever environment and irregular road roughness disturbances actions on the vehicle suspension cause continuous vibration of structure. In previous studies, a stochastic process with a ground displacement power spectral density can be introduced to express the irregular road disturbances as [14], [15]:

$$G_d(\Omega) = \begin{cases} G_d(\Omega_0) \left(\frac{\Omega}{\Omega_0}\right)^{-n_1}, & \Omega \leq \Omega_0 \\ G_d(\Omega_0) \left(\frac{\Omega}{\Omega_0}\right)^{-n_2}, & \Omega > \Omega_0 \end{cases} \quad (1)$$

where $\Omega_0 = 1/2\pi$ is a reference frequency, Ω is a spatial frequency with dimension m^{-1} , which donates the wave numbers per meter. The road roughness constants are presented as n_1 and n_2 , in general, $n_1 = 2$ and $n_2 = 1.5$.

Taken the wheel's characteristic of low pass filtering into consideration, the irregular road disturbances with low frequency could be considered emphatically. Then, the finite sum of Fourier series can approximately estimate the road displacement input $z_r(t)$ from the irregular road disturbances:

$$z_r(t) = \sum_{i=1}^p \xi_i(t) = \sum_{i=1}^p \phi_i \sin(i\omega_0 t + \theta_i), \quad (2)$$

$i = 1, 2, \dots, p,$

where v_0 is the speed of the vehicle, $\omega_0 = 2\pi v_0/l$, l is the length of the road segment, p is used to restrict the range of frequency. $\phi_j = \sqrt{2G_d(j\Delta\Omega)\Delta\Omega}$, $\Delta\Omega = 2\pi/l$, and the initial phase $\theta_j \in [0, 2\pi)$ is a random variable following a uniform disturbance.

In the following, an exosystem is introduced to generate the irregular road roughness disturbances. Denote the following state vectors:

$$\begin{aligned} w(t) &= [w_1(t), \dots, w_{2p}(t)]^T \\ &= [\xi_1(t), \dots, \xi_p(t), \dot{\xi}_1(t), \dots, \dot{\xi}_p(t)]^T \end{aligned} \quad (3)$$

Then, the total irregular road disturbance vector $v(t)$ can be expressed by the following exosystem:

$$\begin{aligned} \dot{w}(t) &= Gw(t), \\ v(t) &= Fw(t), \end{aligned} \quad (4)$$

where

$$\begin{aligned} \bar{G} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \bar{G} & \mathbf{0} \end{bmatrix}, \\ F &= \begin{bmatrix} 0, & \dots, & 0, & \underbrace{1, \dots, 1}_p \end{bmatrix} \\ \bar{G} &= \text{diag} \{ \underbrace{-\omega_0^2, \dots, -\omega_0^2}_p, \underbrace{-(p\omega_0)^2}_p \} \end{aligned} \quad (5)$$

Noting that, the rank of $[F^T \ (FG)^T \ \dots \ (FG^{2p-1})^T]^T = 2p$, the pair (F, \bar{G}) is observable.

It should be noted that the input of irregular road roughness disturbance could be estimated by observer. Therefore, it is necessary to obtain the form of discrete-time system of road roughness disturbances under in-vehicle networks. Let T denote the sampling period, one gets the discrete-time form of Eq. (5)

$$\begin{aligned} w(k+1) &= Gw(k) \\ v(k) &= Fw(k) \end{aligned} \quad (6)$$

where $G = e^{\bar{G}T}$. Assume that (G, F) is completely observable.

III. PROBLEM STATEMENT

In this section, The motion equation of the active vehicle suspension is given. For simplicity, the active vehicle suspension is modeled as a two-degree-freedom quarter-car suspension system. Based on the mechanism of the active vehicle suspension system under in-vehicle networks, the discrete-time models with the fixed sampling period is modeled.

First, consider a simple quarter-car active vehicle suspension system that has been simplified into a TDOF system as shown in Fig. 1.

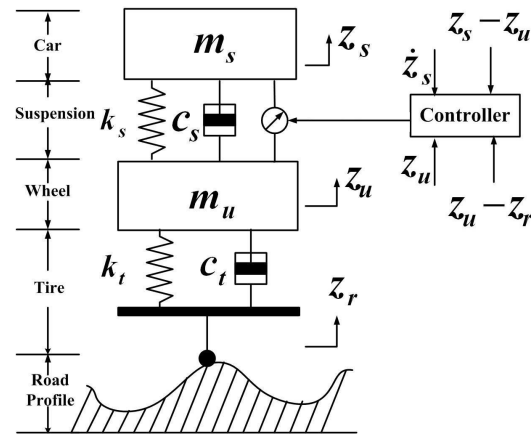


Fig. 1. The Simple Active Suspension System

The motion of simple quarter-vehicle active suspension could be expressed as:

$$\begin{cases} u(t) = m_s \ddot{z}_s(t) + c_s [\dot{z}_s(t) - \dot{z}_u(t)] + k_s [z_s(t) - z_u(t)] \\ -u(t) = m_u \ddot{z}_u(t) + c_s [\dot{z}_u(t) - \dot{z}_s(t)] + k_s [z_u(t) - z_s(t)] \\ k_t [z_u(t) - z_r(t)] + c_t [\dot{z}_u(t) - \dot{z}_r(t)] \end{cases} \quad (7)$$

where m_s is the sprung mass; m_u is the unsprung mass; c_s and k_s are damping and stiffness of the passive suspension system, respectively; k_t and c_t stand for compressibility and damping of the pneumatic tire, respectively; $z_r(t) \in R_q$ is the road displacement input; $u(t)$ represents the active control force of the suspension system. $z_s(t)$ and $z_u(t)$ are the displacements of the unsprung and sprung masses, respectively.

As is well known, the purpose of the active suspension is to minimize the sprung mass acceleration $\ddot{z}_s(t)$, the sprung mass acceleration $\ddot{z}_s(t)$ in the vertical direction, and suspension deflection $z_s(t) - z_u(t)$. In order to satisfy the requirements about ride comfort, road holding ability, and avoidance of rapid deterioration, the controlled output $y_c(t)$ could be defined as:

$$\bar{y}_c(t) = [\ddot{z}_s(t) \quad z_s(t) - z_u(t) \quad z_u(t) - z_r(t)]^T \quad (8)$$

Actually, it is unnecessary and uneconomical to observe full-state of vehicle suspension. Therefore, the measured output $y_m(t)$ is defined as

$$\bar{y}_m(t) = [z_s(t) - z_u(t) \quad \dot{z}_s(t)]^T \quad (9)$$

Then, the variables of system state are defined as:

$$\begin{aligned} \bar{x}_1(t) &= z_s(t) - z_u(t), \bar{x}_2(t) = z_u(t) - z_r(t) \\ \bar{x}_3(t) &= \dot{z}_s(t), \bar{x}_4(t) = \dot{z}_u(t) \end{aligned} \quad (10)$$

where $\bar{x}_1(t)$ is the suspension deflection, $\bar{x}_2(t)$ denotes the tire deflection, $\bar{x}_3(t)$ denotes the speed of sprung mass, and $\bar{x}_4(t)$ is the speed of unsprung mass.

The continuous-time model of simple quarter-car active suspension equation can be described as:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}u(t) + \bar{D}v(t) \\ \bar{y}_c(t) &= \bar{C}\bar{x}(t) + \bar{E}u(t) \\ \bar{y}_m(t) &= \bar{C}\bar{x}(t) \end{aligned} \quad (11)$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ \frac{-k_s}{m_s} & 0 & \frac{-c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & \frac{-k_t}{m_u} & \frac{c_s}{m_u} & \frac{-(c_s+c_t)}{m_u} \end{bmatrix} \\ \bar{B} &= \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_u} \end{bmatrix}, \bar{D} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_u} \end{bmatrix}, \bar{x}(t) = \begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \\ \bar{x}_3(t) \\ \bar{x}_4(t) \end{bmatrix} \\ \bar{C} &= \begin{bmatrix} -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} \frac{1}{m_s} \\ 0 \\ 0 \end{bmatrix} \\ \bar{C} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, v(t) = \dot{z}_r(t). \end{aligned} \quad (12)$$

Consider the mechanism of the simple quarter-car active suspension under in-vehicle networks, the controlled active suspension can be depicted as Fig. 2.

Assume that the delays in ECU-Actuator and Sensor-ECU equal to 0. Then, the discrete-time state space representation of the system (5), obtained with a discretization period T , given by:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Dv(k) \\ y_c(k) &= Cx(k) + Eu(k) \\ y_m(k) &= \bar{C}x(k) \\ x(0) &= x_0 \end{aligned} \quad (13)$$

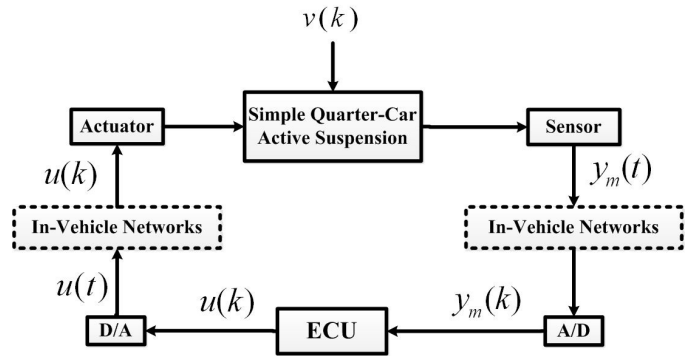


Fig. 2. Active Suspension under in-vehicle networks

where $x(k)$, $u(k)$, $v(k)$ are the state variable, control law, and random road disturbance, respectively, and where $A = e^{\bar{A}T}$, $B = \int_0^T e^{\bar{A}t} \bar{B} dt$, $D = \int_0^T e^{\bar{A}t} \bar{D} dt$.

It should be noted that the irregular road roughness disturbance $v(k)$ will not tend to zero. Then, road disturbance vector $v(k)$, the steady state of the state vector $x(k)$ and the control vector $u(k)$ will not converge to zero synchronously. In this case, the infinite-time average performance index was chosen as:

$$J = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=0}^N [y_c^T(k) Q y_c(k) + u^T(k) R u(k)] \quad (14)$$

where $Q = \text{diag}(q_1, q_2, q_3)$ and R is positive definite matrix.

Then, the objective of this present paper is formulated to find a control $u^*(\cdot)$ to minimize performance index subject to constrains (6) and (13).

IV. FFOVRL LAW

The efficient vibration control algorithm should not only reduce the road roughness disturbances significantly but also be economical. Thus, the FFOVRL is designed to reject the road roughness disturbance. In this section, we will present the detailed FFOVRL algorithm and prove the existence uniqueness conditions of the FFOVRL.

In order to describe the proposed FFOVRL clearly, the matrices are defined as follows:

$$\begin{aligned} \tilde{R} &= R + E^T Q E, A_1 = A - B \tilde{R}^{-1} E^T Q C \\ Q_1 &= C^T Q C - C^T Q E \tilde{R}^{-1} E^T Q C, \end{aligned} \quad (15)$$

Theorem 1: Consider the optimal vibration control problem for active vehicle suspension discrete-time system under in-vehicle networks described (7) and (13) with respect to the quadratic performance indexes (14). The FFOVRL law uniquely exists and can be formulated as:

$$u^*(k) = -\tilde{R}^{-1} \{ B^T A_1^{-T} \bar{P} \omega(k) + [E^T Q C + B^T A_1^{-T} (P - Q_1)] x(k) \} \quad (16)$$

where P is the unique positive definite solution of the following Riccati matrix equation

$$P = Q_1 + A_1^T P S^{-1} A_1 \quad (17)$$

and \bar{P} is the unique solution of the following Stein matrix equation

$$\bar{P} = A_1^T \bar{P} G + A_1^T P S^{-1} (D F - B S^{-1} B^T \bar{P} G) \quad (18)$$

where $S = I + B \tilde{R}^{-1} B^T P$.

Proof: Based on the necessary condition of the maximum principle to Eqs. (6) and (13) with performance index (14), the FFOVRL law can be described by

$$u^*(k) = -\tilde{R}^{-1} [E^T Q C x(k) + B^T \lambda(k+1)] \quad (19)$$

where $\lambda(k)$ is the solution of the following two-point boundary value (TPBV) problem

$$\begin{cases} x(k+1) = A_1 x(k) - B \tilde{R}^{-1} B^T \lambda(k+1) + D F \omega(k) \\ \lambda(k) = Q_1 x(k) + A_1^T \lambda(k+1) \end{cases} \quad (20)$$

To solve the TPBV problem (20), let

$$\lambda(k) = P x(k) + \bar{P} \omega(k) \quad (21)$$

Based on the Eqs. (20) and (21), one obtains

$$\begin{cases} \lambda(k+1) = A_1^{-T} [(P - Q_1) x(k) + \bar{P} \omega(k)] \\ x(k+1) = \\ S^{-1} [A_1 x(k) + (D F - B \tilde{R}^{-1} B^T \bar{P} G) \omega(k)] \end{cases} \quad (22)$$

Substituting the first formula of Eq. (22) into (19), the FFOVC law (16) is obtained. Rearranging Eqs. (20) and (22), one gets

$$\lambda(k) = [Q_1 + A_1^T P S^{-1} A_1] x(k) + [A_1^T \bar{P} G + A_1^T P S^{-1} (D F - B S^{-1} B^T \bar{P} G)] \omega(k) \quad (23)$$

By comparing the coefficients of Eqs. (21) and (23), the Riccati matrix Eq. (17) and Stein matrix Eq. (18) are obtained.

It should be noted that the pair (A, B) is controllable and the pair (A, C) is observable. Therefore, the matrix P is the unique positive definite solution of Eq. (17) [16]. From Eqs. (6) and the second formula of Eq. (22), one gets

$$|\lambda_i(S^{-1} A_1)| \times |\lambda_j(G)| < 1, \quad (24)$$

$i = 1, 2, \dots, n; j = 1, 2, \dots, 2p;$

Then, \bar{P} is existent and unique solution of Stein matrix equation (18) [17]. When P and \bar{P} are derived, $\lambda(k)$ and the FFOVRL $u^*(k)$ could be obtain from Eqs. (16) and (21), respectively. Then, the existent and unique of the FFOVRL (16) are proved. The proof is completed.

V. SIMULATION

In this section, simulation experiments are shown to illustrate the effectiveness of the FFOVRL for the active suspension systems.

The road displacement input is estimated as a random process with a ground displacement power spectral density of (1), in which the parameters of road roughness disturbance is selected shown in Table I. Consequently, the road displacement input $z_r(t)$ is displayed in Fig. 3.

The parameters of vehicle active suspension system model are shown in Table II [18]. It should be noted that the dimension of the control force is N .

TABLE I
THE PARAMETERS OF ROAD ROUGHNESS DISTURBANCE

Coefficient	Value
$G_d(\Omega_0)$	$256 \times 10^{-6} \text{m}^3$
n_1	2
n_2	1.5
ω_1	3.3144
ω_2	33.1440

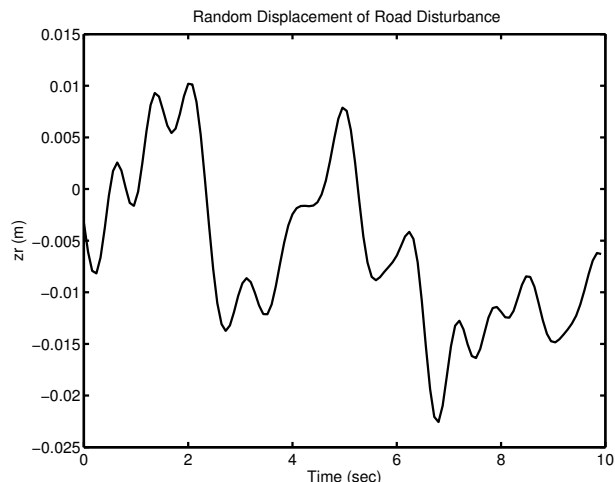


Fig. 3. Random Displacement of Road Disturbances

Then, under the in-vehicle networks with the sampling period $T = 0.08s$, the matrix values of the discrete-time active suspension system (13) are given by

$$A = \begin{bmatrix} 0.8754 & 0.2627 & 0.0734 & -0.0661 \\ 0.1110 & 0.7358 & 0.0059 & 0.0668 \\ -0.3290 & -0.0628 & 0.9779 & 0.0206 \\ 2.5350 & -6.0630 & 0.1760 & 0.5589 \end{bmatrix}, \quad (25)$$

$$B = \begin{bmatrix} 0.0292 \\ -0.0260 \\ 0.0770 \\ -0.5934 \end{bmatrix} \times 10^{-4}, \quad D = \begin{bmatrix} -0.0073 \\ -0.0727 \\ 0.0015 \\ 0.2651 \end{bmatrix},$$

Assume that sprung mass acceleration $\ddot{z}_s(t)$, suspension deflection $z_s(t) - z_u(t)$ and tire deflection $z_u(t) - z_r(t)$ are of equal importance in ride comfort. So we select $q_1 = q_2 = q_3 = 10^6$ in performance index (14).

The curves of sprung mass acceleration, suspension deflection, and tyre deflection are presented in Figs. 3-5, respectively. The comparison results between FFOVRL law and open-loop system are shown. Also, the root-mean-square (RMS) values of

TABLE II
THE PARAMETERS OF VEHICLE ACTIVE SUSPENSION

Coefficient	Value	Unit
Sprung mass m_s	180	kg
Unsprung mass m_u	25	kg
Stiffness k_s	16000	N/m
Compressibility of the pneumatic tire k_t	190000	N/m
Damping of the active suspension system b_s	1000	N/m

TABLE III
COMPARISON OF RMS VALUES OF PERFORMANCE CRITERIA

Coefficient	$\ddot{z}_s(m^2/s)$	$z_s - z_u(m)$	$z_u - z_r(m)$
Optimal Control	0.2832	0.0492	0.0242
Open-Loop	0.7003	0.1732	0.0730
Reduced Rate (%)	59.22	71.59	66.84

performance index are presented in Table III for FFOVRL and open-loop system, which include sprung mass acceleration, suspension deflection, and tyre deflection.

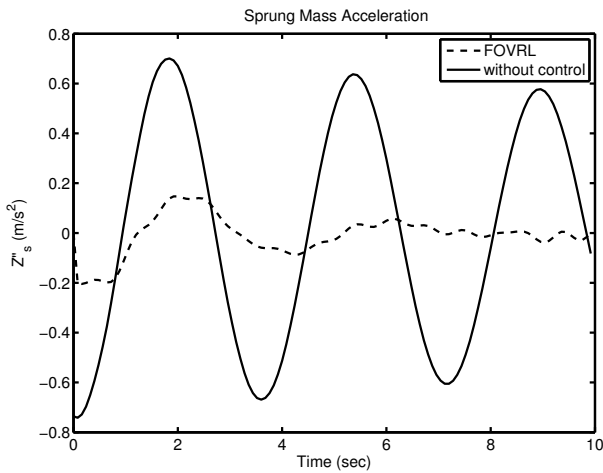


Fig. 4. The Curves of Sprung Mass Acceleration

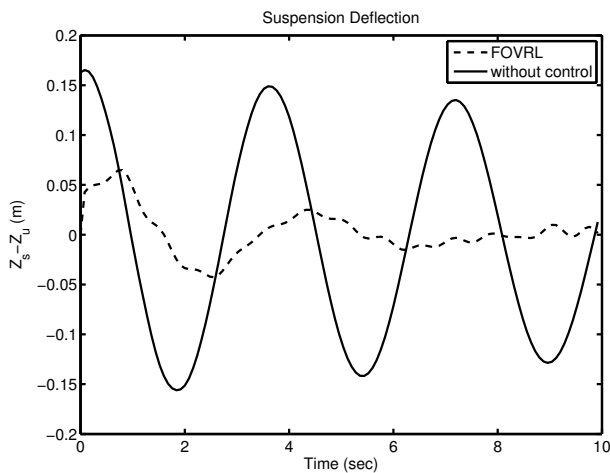


Fig. 5. The Curves of Suspension Deflection

From Figs. 4-6 and Table III, it can be clearly seen that the vibration process of the active suspension is significantly reduced by the FFOVRL. The simulation result depicted in Table 3 shows that the performance index is much better than open-loops. Then, the irregular road roughness disturbances could be rejected significantly by using FFOVRL. Also, the presented FFOVRL is economical.

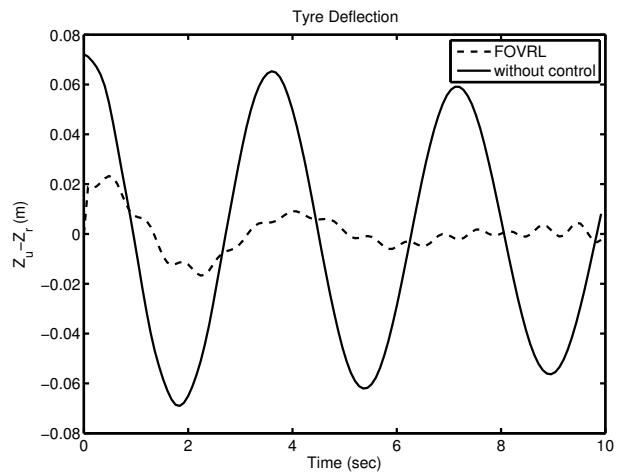


Fig. 6. The Curves of Tyre Deflection

VI. CONCLUSION

This paper has been concerned with the development of optimal vibration control for the vehicle active suspension subject to road roughness disturbances under in-vehicle networks. The improvements in design of exosystem for producing the irregular road roughness disturbances, and the control algorithm implementation are presented under in-vehicle networks. The exosystems is determined by the dynamic equation of the irregular road disturbances. Another significant improvement is on the FFOVRL. FFOVRL can reduce the vibration significantly and keep economical in an optimal fashion under in-vehicle networks. A numerical example of the FFOVRL for an active vehicle suspension is studied.

ACKNOWLEDGMENT

This work was partially supported by the Program for Scientific research innovation team in Colleges and Universities of Shandong Province (TTD1202), the National Natural Science Foundation of China under Grant (61074092, 61070130, 61203105), the Natural Science Foundation of Shandong Province under Grant (ZR2010FM019, ZR2011FZ003, ZR2012FQ016), the framework of the IT4 Innovations Centre of Excellence project, reg. no. CZ.1.05/1.1.00/02.0070 by operational programme Research and Development for Innovations funded by the Structural Funds of the European Union and state budget of the Czech Republic, EU, and the Doctoral Foundation of University of Jinan under Grant (XBS1318).

REFERENCES

- [1] J. Roshan, K. Shankar, "Modeling and optimization of passive and semi-active suspension systems for passenger cars to improve ride comfort and isolate engine vibration," *Journal of Vibration and Control*, vol. 19, no. 10, pp. 1471-1479, 2013
- [2] J. Z. Jiang, A. Z. Matamoros-Sanchez, R. M. Roger, M. C. Smith, "Passive suspensions incorporating inerters for railway vehicles," *Vehicle System Dynamics*, vol. 50, no. 1, pp. 263-276, 2012
- [3] D. Hrovat, "Optimal active suspension structures for quarter-car vehicle models", *Automatica*, vol. 26, pp. 845-860, 1990.

- [4] S. Young, S. Vladimir, "Distributed mixed continuous-discrete receding horizon filter for multisensory uncertain active suspension systems with measurement delays", IET Control Theory and Applications, vol. 7, no. 15, pp. 1922-1031, 2013
- [5] M. M. Ma and H. Chen, "Disturbance attenuation control of active suspension with non-linear actuator dynamics", IET Control Theory and Applications, vol. 5, pp. 112-122, Feb 2011.
- [6] H. Gao, W. Sun, and P. Shi, "Robust sampled-data H_∞ control for vehicle active suspension systems", IEEE Transaction on Control Systems Technology, vol. 18, pp. 238-245, Feb 2010.
- [7] D. Hrovat, "Survey of advanced suspension developments and related optimal control applications", Automatica, vol. 33, pp. 1781-1817, 1997.
- [8] H. Du and N. Zhang, " H_∞ control of active vehicle suspensions with actuator time delay", Journal of Sound Vibration, vol. 301, No. 1-2, pp. 236-252, 2007.
- [9] Y. M. Sam and J. H. S. B. Osman, "Modeling and control of the active suspension system using proportional integral sliding mode approach", Asian Journal of Control, vol. 7, pp. 91-98, 2005.
- [10] W. Sun, H. Gao, O. Kaynak, "Adaptive backstepping control for active suspension systems with hard constraints", IEEE Transactions on Mechatronics, vol. 8, no. 3, pp. 1073-1079, 2013.
- [11] F. M. Raimondi and M. Melluso, "Fuzzy motion control strategy for cooperation of multiple automated vehicles with passengers comfort", Automatica, vol. 44, pp. 2804-2816, Nov 2008.
- [12] L. Jing, "Suboptimal vibration control for nonlinear suspension systems based on in-vehicle networks", Proceeding of 2011 International Conference on System Science and Engineering, pp. 239-244, 2011.
- [13] S. Y. Han, G. Y. Tang, and Y. H. Chen, et al., "Optimal vibration control for vehicle active suspension discrete-time systems with actuator time delay", Asian Journal of Control, vol. 15, pp. 1579-1588, 2013.
- [14] G. Verros, S. Natsiavas, and C. Papadimitriou, "Design optimization of quarter-car models with passive and semi-active suspensions under random road excitation", Journal of Sound Vibration, vol. 11, pp. 581-606, 2005.
- [15] Y. C. Zhang, L. X. Wang, and H. Cong H, "Present situation and future challenges for automobile active suspension control", Control Theory and Applications, vol. 21, PP. 139-144, 2004.
- [16] S. Y. Han, G. Y. Tang, C. M. Zhang, "Near-optimal tracking control for discrete-time systems with delayed input", International Journal of Control, Automation, and Systems, vol. 8, no. 6, pp. 1330-1335, 2010.
- [17] P. Lancaster, L. Lerer, and M. Tismenetsky, "Factored forms for solutions of $AX - XB = C$ and $X - AXB = C$ in companion matrices", Linear Algebra and Its Applications, vol. 62, pp.19-49, 1984
- [18] G. Priyandoko, M. Mailah, and H. Jamaluddin, "Vehicle active suspension system using skyhook adaptive neuro active force control", Mechanical Systems and Signal Processing, vol. 23, pp. 855-868, 2009.