Differential Evolution Using a Neighborhood-based Mutation Operator

Swagatam Das[1], Ajith Abraham[2], Uday K. Chakraborty[3], and Amit Konar[1]

[1]Department of Electronics and Telecommunication Engineering, Jadavpur University, Kolkata 700032, India
[2]Center of Excellence for Quantifiable Quality of Service, Norwegian University of Science and Technology, Norway
[3]Dept. of Math & Comp. Sc. University of Missouri, St. Louis, MO 63121, USA
swagatamdas19@yahoo.co.in, ajith.abraham@ieee.org, chakrabortyu@umsl.edu, konaramit@yahhoo.co.in

Abstract- Differential Evolution (DE) is well known as a simple and efficient scheme for global optimization over continuous spaces. It has reportedly outperformed a few Evolutionary Algorithms (EAs) and other search heuristics like the Particle Swarm Optimization (PSO) when tested over both benchmark and real-world problems. DE, however, is not completely free from the problems of slow and/or premature convergence. This article describes a family of improved variants of the DE/target-to-best/1/bin scheme, which utilize the concept of the neighborhood of each population member. The idea of small neighborhoods, defined over the index-graph of parameter vectors, draws inspiration from the community of the PSO algorithms. The proposed schemes balance the exploration and exploitation abilities of DE without imposing serious additional burdens in terms of function evaluations. They are shown to be statistically significantly better than or at least comparable to several existing DE variants as well as a few other significant evolutionary computing techniques over a test-suite of 24 benchmark functions. The paper also investigates the applications of the new DE-variants to two real-life problems concerning parameter estimation for frequency modulated sound waves and spread spectrum radar poly-phase code design.

Keywords: Differential evolution, Evolutionary algorithms, Numerical optimization, Particle swarm optimization, Meta-heuristics.

1 Introduction

Differential Evolution (DE), proposed by Storn and Price [1-3], is a simple yet powerful algorithm for real parameter optimization. Recently, the DE algorithm has become quite popular in the machine intelligence and cybernetics communities. It has successfully been applied to diverse domains of science and engineering, such as mechanical engineering design [4, 5], signal processing [6], chemical engineering [7, 8], machine intelligence, and pattern recognition [9, 10]. It has been shown to perform better than the Genetic Algorithm (GA) [11] or the Particle Swarm Optimization (PSO) [12] over several numerical benchmarks [13]. Many of the most recent developments in DE algorithm design and applications can be found in [14]. Like other evolutionary algorithms, two fundamental processes drive the evolution of a DE population: the *variation* process, which enables exploring different regions of the search space and the *selection* process, which ensures the exploitation of previous knowledge about the fitness landscape.

Practical experience, however, shows that DE may occasionally stop proceeding toward the global optimum even though the population has not converged to a local optimum or any other point [15]. Occasionally, even new individuals may enter the population, but the algorithm does not progress by finding any better solutions. This situation is usually referred to as *stagnation*. DE also suffers from the problem of premature convergence, where the population converges to some local optima of a multi-modal objective function, losing its diversity. The probability of stagnation depends on how many different potential trial solutions are available and also on their capability to enter into the population of the subsequent generations [15]. Like other evolutionary computing algorithms, the performance of DE deteriorates with the growth of the dimensionality of the search space as well. There exists a good volume of works (a review of which can be found in Section 3), attempting to improve the convergence speed and robustness (ability to produce similar results over repeated runs) of DE by tuning the parameters like population size *NP*, the scale factor *F*, and the crossover rate *Cr*.

In the present work, we propose a family of variants of the DE/target-to-best/1 scheme [3, page 140], which was also referred to as "Scheme DE2" in the first technical paper on DE [1]. In some DE literature this algorithm is referred to as DE/current-to-best/1 [16, 17]. To combine the exploration and exploitation capabilities of DE, we propose a new hybrid mutation scheme that utilizes an explorative and an exploitive mutation operator, with an objective of balancing their effects. The explorative mutation operator (referred to as the *local* mutation model) has a greater possibility of locating the minima of the objective function, but generally needs more iterations (generations). On the other hand, the exploitative mutation operator (called by us the *global* mutation model) rapidly converges to a minimum of the objective function. In this case there exists the danger of premature convergence to a suboptimal solution. In the hybrid model we linearly combine the two mutation operators using a new parameter, called the weight factor. Four different schemes have been proposed and investigated for adjusting the weight factor, with a view to alleviating user intervention and hand tuning as much as possible.

Here we would like to mention that although a preliminary version of this work appeared as a conference paper in [18], the present version has been considerably enhanced and it differs in many aspects from [18]. It critically examines the effects of the global and local neighborhoods on the performance of DE and explores a few different ways of tuning of the weight factor (see Section 4) used for unification of the neighborhood models. In addition, it compares the performance of the proposed approaches with several state-of-the-art DE variants as well as other evolutionary algorithms over a test-bed of 24 well-known numerical benchmarks and one real-world optimization problem in contrast to [18], which uses only six benchmarks and provides limited comparison results.

The remainder of this paper is organized as follows. In Section 2, we provide a brief outline of the DE family of algorithms. Section 3 provides a short survey of previous research on improving the performance of DE. Section 4 introduces the proposed family of variants of the DE/target-to-best/1 algorithm. Experimental settings for the benchmarks and simulation strategies are explained in Section 5. Results are presented and discussed in Section 6. Finally, conclusions are drawn in Section 7.

2 The DE Algorithm

Like any other evolutionary algorithm, DE starts with a population of *NP* *D*-dimensional parameter vectors representing the candidate solutions. We shall denote subsequent generations in DE by $G = 0,1...,G_{\max}$. Since the parameter vectors are likely to be changed over different generations, we may adopt the following notation for representing the *i*-th vector of the population at the current generation as:

$$\vec{X}_{i,G} = [x_{1,i,G}, x_{2,i,G}, x_{3,i,G}, ..., x_{D,i,G}]. \tag{1}$$

For each parameter of the problem, there may be a certain range within which the value of the parameter should lie for better search results. The initial population (at $G = 0$) should cover the entire search space as much as possible by uniformly randomizing individuals within the search space constrained by the prescribed minimum and maximum bounds: $\vec{X}_{\min} = \{x_{1,\min}, x_{2,\min}, ..., x_{D,\min}\}$ and $\vec{X}_{\max} = \{x_{1,\max}, x_{2,\max}, ..., x_{D,\max}\}$. Hence we may initialize the *j*-th component of the *i*-th vector as

$$x_{j,i,0} = x_{j,\min} + rand_{i,j}(0,1) \cdot (x_{j,\max} - x_{j,\min}) \,, \quad (2)$$

where $rand_{i,j}(0,1)$ is a uniformly distributed random number lying between 0 and 1 and is instantiated independently for each component of the *i*-th vector. The following steps are taken next: mutation, crossover, and selection (in that order), which are explained in the following subsections.

a) Mutation:

After initialization, DE creates a *donor* vector $\vec{V}_{i,G}$ corresponding to each population member or *target* vector $\vec{X}_{i,G}$ in the current generation through mutation and sometimes using arithmetic recombination too. It is the method of creating this donor vector that differentiates one DE scheme from another. Five most frequently referred strategies, implemented in the public-domain DE codes for producing the donor vectors (available online at http://www.icsi.berkeley.edu/~storn/code.html) are listed below:

"DE/rand/1": $\vec{V}_{i,G} = \vec{X}_{r_1^i,G} + F \cdot (\vec{X}_{r_2^i,G} - \vec{X}_{r_3^i,G})$. (3)

"DE/best/1": $\vec{V}_{i,G} = \vec{X}_{best,G} + F \cdot (\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G})$. (4)

"DE/target-to-best/1": $\vec{V}_{i,G} = \vec{X}_{i,G} + F \cdot (\vec{X}_{best,G} - \vec{X}_{i,G}) + F \cdot (\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G})$. (5)

"DE/best/2": $\vec{V}_{i,G} = \vec{X}_{best,G} + F \cdot (\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G}) + F \cdot (\vec{X}_{r_3^i,G} - \vec{X}_{r_4^i,G})$. (6)

"DE/rand/2": $\vec{V}_{i,G} = \vec{X}_{r_1^i,G} + F \cdot (\vec{X}_{r_2^i,G} - \vec{X}_{r_3^i,G}) + F \cdot (\vec{X}_{r_4^i,G} - \vec{X}_{r_5^i,G})$. (7)

The indices $r_1^i, r_2^i, r_3^i, r_4^i$, and r_5^i are mutually exclusive integers randomly chosen from the range [1, *NP*], and all are different from the base index *i*. These indices are randomly generated once for each donor vector. The scaling factor *F* is a positive control parameter for scaling the difference vectors. $\vec{X}_{best,G}$ is the best individual vector with the best fitness (i.e. lowest objective function value for a minimization problem) in the population at generation *G*. Note that some of the strategies for creating the donor vector may be mutated recombinants, for example, equation (5) listed above basically mutates a two-vector recombinant: $\vec{X}_{i,G} + F \cdot (\vec{X}_{best,G} - \vec{X}_{i,G})$. The general convention used for naming the various mutation strategies is DE/x/y/z, where DE stands for Differential Evolution, x represents a string denoting the vector to be perturbed, y is the number of difference vectors considered for perturbation of x, and z stands for the type of crossover being used (exp: exponential; bin: binomial). The following section discusses the crossover step in DE.

b) Crossover:

To increase the potential diversity of the population, a crossover operation comes into play after generating the donor vector through mutation. The DE family of algorithms can use two kinds of crossover schemes - *exponential* and *binomial* [1-3]. The donor vector exchanges its components with the target vector $\vec{X}_{i,G}$ under this operation to form the *trial* vector $\vec{U}_{i,G} = [u_{1,i,G}, u_{2,i,G}, u_{3,i,G}, \ldots, u_{D,i,G}]$. In exponential crossover, we first choose an integer *n* randomly among the numbers $[1, D]$. This integer acts as a starting point in the target vector, from where the crossover or exchange of components with the donor vector starts. We also choose another integer *L* from the interval $[1, D]$. *L* denotes the number of components; the donor vector actually contributes to the target. After a choice of *n* and *L* the trial vector is obtained as:

$$u_{j,i,G} = \begin{cases} v_{j,i,G}, & \text{for } j = \langle n \rangle_D, \langle n+1 \rangle_D, \ldots, \langle n+L-1 \rangle_D \\ x_{j,i,G}, & \text{for all other } j \in [1, D], \end{cases} \tag{8}$$

where the angular brackets $\langle\ \rangle_D$ denote a modulo function with modulus *D*. The integer *L* is drawn from $[1, D]$ according to the following pseudo-code.

```
$L = 0$;
DO
{
   $L = L+1$;
} WHILE (( $(rand(0,1) < Cr)$ AND ( $L < D$ ));
```

'*Cr*' is called the *crossover rate* and appears as a control parameter of DE just like *F*. Hence in effect, probability $(L \geq \upsilon) = (Cr)^{\upsilon-1}$ for any $\upsilon > 0$. For each donor vector, a new set of *n* and *L* must be chosen randomly as shown above.

On the other hand, binomial crossover is performed on each of the *D* variables whenever a randomly picked number between 0 and 1 is less than or equal to the *Cr* value. In this case the number of parameters inherited from the donor has a (nearly) binomial distribution. The scheme may be outlined as:

$$u_{j,i,G} = \begin{cases} v_{j,i,G}, & \text{if } (rand_{i,j}(0,1) \leq Cr \text{ or } j = j_{rand}) \\ x_{j,i,G}, & \text{otherwise,} \end{cases} \tag{9}$$

where $rand_{i,j}(0,1) \in [0,1]$ is a uniformly distributed random number, which is called anew for each *j*-th component of the *i*-th parameter vector. $j_{rand} \in [1,2,....,D]$ is a randomly chosen index, which ensures that $\vec{U}_{i,G}$ gets at least one component from $\vec{V}_{i,G}$.

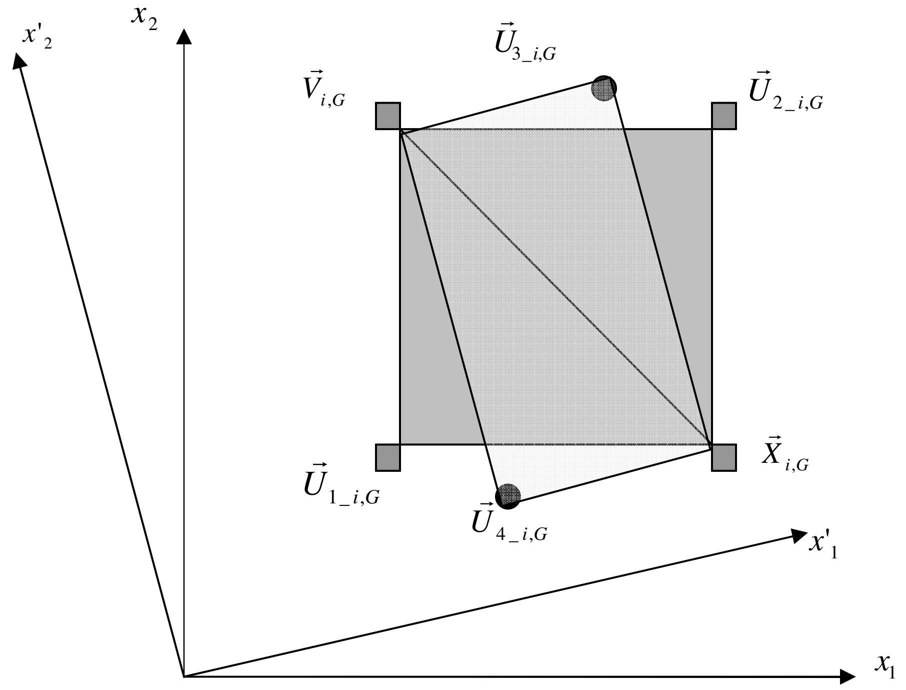


Fig. 1. Change of the trial vectors generated through the crossover operation described in equation (9) due to rotation of the coordinate system.

The crossover operation described in equation (9) is basically a discrete recombination [3]. Figure 1 illustrates a two-dimensional example of recombining the parameters of two vectors $\vec{X}_{i,G}$ and $\vec{V}_{i,G}$, according to this crossover operator, where the potential trial vectors are generated at the corners of a rectangle. Note that $\vec{V}_{i,G}$ can itself be the trial vector (i.e. $\vec{U}_{i,G}=\vec{V}_{i,G}$) when $Cr = 1$. As can be seen from Figure 1, discrete recombination is a rotationally variant operation. Rotation transforms the coordinates of both vectors and thus changes the shape of the rectangle as shown in Figure 1. Consequently, the potential location of the trial vector moves from the possible set ($\vec{U}_{1\_i,G}, \vec{U}_{2\_i,G}$) to ($\vec{U}_{3\_i,G}, \vec{U}_{4\_i,G}$). To overcome this limitation, a new trial vector generation strategy 'DE/current-to-rand/1' is proposed in [19], which replaces the crossover operator prescribed in equation (9) with the rotationally invariant arithmetic crossover operator to generate the trial vector $\vec{U}_{i,G}$ by linearly combining the target vector $\vec{X}_{i,G}$ and the corresponding donor vector $\vec{V}_{i,G}$ as follows:

$$\vec{U}_{i,G} = \vec{X}_{i,G} + K \cdot (\vec{V}_{i,G} - \vec{X}_{i,G}).$$

Now incorporating equation (3) in equation (10) we have:

$$\vec{U}_{i,G} = \vec{X}_{i,G} + K \cdot (\vec{X}_{r_1,G} + F \cdot (\vec{X}_{r_2,G} - \vec{X}_{r_3,G}) - \vec{X}_{i,G}),$$

which further simplifies to:

$$\vec{U}_{i,G} = \vec{X}_{i,G} + K \cdot (\vec{X}_{r_1,G} - \vec{X}_{i,G}) + F^{/} \cdot (\vec{X}_{r_2,G} - \vec{X}_{r_3,G}), \tag{10}$$

where K is the combination coefficient, which has been shown [19] to be effective when it is chosen with a uniform random distribution from [0, 1] and $F^{/} = K \cdot F$ is a new constant here.

c) Selection:

To keep the population size constant over subsequent generations, the next step of the algorithm calls for *selection* to determine whether the target or the trial vector survives to the next generation i.e. at $G = G + 1$. The selection operation is described as:

$$\begin{aligned} \vec{X}_{i,G+1} &= \vec{U}_{i,G}, \quad \text{if } f(\vec{U}_{i,G}) \le f(\vec{X}_{i,G}) \\ &= \vec{X}_{i,G}, \quad \text{if } f(\vec{U}_{i,G}) > f(\vec{X}_{i,G}), \end{aligned} \tag{11}$$

where $f(\vec{X})$ is the function to be minimized. So if the new trial vector yields an equal or lower value of the objective function, it replaces the corresponding target vector in the next generation; otherwise the target is retained in the population. Hence the population either gets better (with respect to the minimization of the objective function) or remains the same in fitness status, but never deteriorates. The complete pseudo-code of the DE is given below:

Pseudo-code for the DE algorithm family

```
Step 1. Set the generation number $G = 0$ and randomly initialize a population of $NP$
        individuals $P_G = \{\vec{X}_{1,G},......,\vec{X}_{NP,G}\}$ with $\vec{X}_{i,G} = [x_{1,i,G}, x_{2,i,G}, x_{3,i,G},....., x_{D,i,G}]$ and each
        individual uniformly distributed in the range $[\vec{X}_{\min}, \vec{X}_{\max}]$,
        where $\vec{X}_{\min} = \{x_{1,\min}, x_{2,\min},..., x_{D,\min}\}$ and $\vec{X}_{\max} = \{x_{1,\max}, x_{2,\max},..., x_{D,\max}\}$ with
        $i = [1,2,...., NP]$.

Step 2. WHILE the stopping criterion is not satisfied
        DO
```

```
    FOR $i = 1$ to $NP$                                //do for each individual sequentially
      **Step 2.1** *Mutation Step*
        Generate a donor vector $\vec{V}_{i,G} = \{v_{1,i,G}, \ldots\ldots., v_{D,i,G}\}$ corresponding to the $i$-th target
        vector $\vec{X}_{i,G}$ via one of the different mutation schemes of DE (equations (3) to (7))
      **Step 2.2** *Crossover Step*
        Generate a trial vector $\vec{U}_{i,G} = \{u_{1,i,G}, \ldots\ldots., u_{D,i,G}\}$ for the $i$-th target vector
        $\vec{X}_{i,G}$ through binomial crossover (equation (9)) or exponential crossover (equation (8))
        or through the arithmetic crossover (equation (10)).
      **Step 2.3** *Selection Step*
        Evaluate the trial vector $\vec{U}_{i,G}$
        IF $f(\vec{U}_{i,G}) \le f(\vec{X}_{i,G})$, THEN $\vec{X}_{i,G+1} = \vec{U}_{i,G}$, $f(\vec{X}_{i,G+1}) = f(\vec{U}_{i,G})$
          IF $f(\vec{U}_{i,G}) < f(\vec{X}_{best,G})$, THEN $\vec{X}_{best,G} = \vec{U}_{i,G}$, $f(\vec{X}_{best,G}) = f(\vec{U}_{i,G})$
          END IF
        END IF
        ELSE $\vec{X}_{i,G+1} = \vec{X}_{i,G}$, $f(\vec{X}_{i,G+1}) = f(\vec{X}_{i,G})$
    END FOR
    **Step 2.4** Increase the Generation Count $G = G + 1$
END WHILE
```

3 A Review of Previous Work on Improving the DE Algorithm

Over the past few years researchers have been investigating ways of improving the ultimate performance of the DE algorithm by tuning its control parameters. Storn and Price in [1] have indicated that a reasonable value for *NP* could be between 5*D* and 10*D* (*D* being the dimensionality of the problem), and a good initial choice of *F* could be 0.5. The effective value of *F* usually ranges in [0.4, 1].

Gamperle *et al.* [20] evaluated different parameter settings for DE on the Sphere, Rosenbrock's and Rastrigin's functions. Their experimental results revealed that the global optimum searching capability and the convergence speed are very sensitive to the choice of control parameters *NP, F*, and *Cr*. Furthermore, a plausible choice of the population size *NP* is between 3*D* and 8*D*, with the scaling factor *F* = 0.6 and the crossover rate *Cr* in [0.3, 0.9]. Recently, the authors in [16] claim that typically 0.4 < *F* < 0.95 with *F* = 0.9 is a good first choice. *Cr* typically lies in (0, 0.2) when the function is separable, while in (0.9, 1) when the function's parameters are dependent.

As can be seen from the literature, several claims and counter-claims were reported concerning the rules for choosing the control parameters, confusing engineers, who try to solve real-world optimization problems with DE. Further, many of these claims lack sufficient experimental justification. Therefore researchers consider techniques such as self-adaptation to avoid manual tuning of the parameters of DE. Usually self-adaptation is applied to tune the control parameters *F* and *Cr*. Liu and Lampinen introduced Fuzzy Adaptive Differential Evolution (FADE) [21] using fuzzy logic controllers, whose inputs incorporate the relative function values and individuals of successive generations to adapt the parameters for the mutation and crossover operation. Based on the experimental results over a set of benchmark functions, the FADE algorithm outperformed the conventional DE algorithm. In this context, Qin *et al.* proposed a Self-adaptive DE (SaDE) [22] algorithm, in which both the trial vector generation strategies and their associated parameters are gradually self-adapted by learning from their previous experiences of generating promising solutions.

Zaharie proposed a parameter adaptation strategy for DE (ADE) based on the idea of controlling the population diversity, and implemented a multi-population approach [23]. Following the same line of thinking, Zaharie and Petcu designed an adaptive Pareto DE algorithm for multi-objective optimization and also analyzed its parallel implementation [24]. Abbass [25] self-adapted the crossover rate *Cr* for multi-objective optimization problems, by encoding the value of *Cr* into each individual and simultaneously evolving it with other search variables. The scaling factor *F* was generated for each variable from a Gaussian distribution *N* (0, 1).

Omran *et al.* [26] introduced a self-adaptive scaling factor parameter *F*. They generated the value of *Cr* for each individual from a normal distribution *N* (0.5, 0.15). This approach (called SDE) was tested on four benchmark functions and performed better than other versions of DE. Besides adapting the control parameters *F* or *Cr*, some researchers also adapted the population size. Teo proposed DE with Self Adapting Populations (DESAP) [27], based on Abbass's self-adaptive Pareto DE [25]. Recently, Brest *et al.* [28] encoded control parameters *F* and *Cr* into the individual and evolved their values by using two new probabilities τ_1 and τ_2. In their algorithm (called SADE), a set of *F* values was assigned to each individual in the population. With probability τ_1, *F* is reinitialized to a new random value in the range of [0.1, 1.0], otherwise it is kept unchanged. The control parameter *Cr*, assigned to each individual, is adapted in an identical fashion, but with a different re-initialization range of [0, 1] and with the probability τ_2. With probability τ_2, *Cr* takes a random value in [0, 1], otherwise it retains its earlier value in the next generation.

Das *et al.* [29] introduced two schemes for adapting the scale factor *F* in DE. In the first scheme (called DERSF: DE with Random Scale Factor) they varied *F* randomly between 0.5 and 1.0 in successive iterations. They suggested decreasing *F* linearly from 1.0 to 0.5 in their second scheme (called DETVSF: DE with Time varying Scale Factor). This encourages the individuals to sample diverse zones of the search space during the early stages of the search. During the later stages, a decaying scale factor helps to adjust the movements of trial solutions finely so that they can explore the interior of a relatively small space in which the suspected global optimum lies.

DE/rand/1/either-or is a state-of-the-art DE variant described by Price *et al.* [3, page 118]. In this algorithm, the trial vectors that are pure mutants occur with a probability p_F and those that are pure recombinants occur with a probability $1 - p_F$. The scheme for trial vector generation may be outlined as:

$$\begin{aligned}\vec{U}_{i,G} &= \vec{X}_{r_1^i,G} + F \cdot (\vec{X}_{r_2^i,G} - \vec{X}_{r_3^i,G}), \qquad \text{if } rand_i(0,1) < p_F \\ &= \vec{X}_{r_1^i,G} + K \cdot (\vec{X}_{r_2^i,G} + \vec{X}_{r_3^i,G} - 2.\vec{X}_{r_1^i,G}), \text{ otherwise} \end{aligned} \tag{12}$$

Where, according to Price *et al.*, $K = 0.5 \cdot (F + 1)$ serves as a good choice of the parameter *K* for a given *F*.

Rahnamayan *et al.* have proposed an Opposition-based DE (ODE) [30] that is specially suited for noisy optimization problems. The conventional DE algorithm was enhanced by utilizing the opposition number-based optimization concept in three levels, namely, population initialization, generation jumping, and local improvement of the population's best member.

Yang *et al.* [31] proposed a hybridization of DE with the Neighborhood Search (NS), which appears as a main strategy underpinning Evolutionary Programming (EP) [32]. The resulting algorithm, known as NSDE, performs mutation by adding a normally distributed random value to each target-vector component in the following way:

$$\vec{V}_{i,G} = \vec{X}_{r_1^i,G} + \begin{cases} \vec{d}_{i,G}.N(0.5,0.5)\text{, if } rand_i(0,1) < 0.5 \\ \vec{d}_{i,G}.\delta\text{,} \qquad \text{otherwise,} \end{cases} \tag{13}$$

where $\vec{d}_{i,G} = \vec{X}_{r_2^i,G} - \vec{X}_{r_3^i,G}$ is the usual difference vector, *N*(0.5, 0.5) denotes a Gaussian random number with mean 0.5 and standard deviation 0.5, and δ denotes a Cauchy random variable with scale parameter $t = 1$. Recently Yang *et al.* [33] used a Self-adaptive NSDE in the cooperative coevolution framework that is capable of optimizing large scale non-separable problems (up to 1000 dimensions). They proposed a random grouping scheme and adaptive weighting for problem decomposition and coevolution. Somewhat similar in spirit to the present paper is the study by Yang *et al.* [34] on self-adaptive differential evolution with neighborhood search (SaNSDE). SaNSDE incorporates self-adaptation ideas from the Qin *et al*'s SaDE [22] and proposes three self-adaptive strategies: self-adaptive choice of the mutation strategy between two alternatives, self-adaptation of the scale factor *F*, and self-adaptation of the crossover rate *Cr*. We would like to point out here that in contrast to Yang *et al.*'s works on NSDE and SaNSDE, we keep the scale factor non-random and use a ring-shaped neighborhood topology (inspired by PSO [37]), defined on the index graph of the parameter vectors, in order to derive a local neighborhood-based mutation model. Also instead of *F* and *Cr*, the weight factor that unifies two kinds of mutation models, have been made self-adaptive in one of the variants of DE/target-to-best/1 scheme, proposed by us. Section 4 describes these issues in sufficient details.

Noman and Iba [35, 36] proposed the *Fittest Individual Refinement* (FIR); a crossover-based local search method for DE. The FIR scheme accelerates DE by enhancing its search capability through exploration of the neighborhood of the best solution in successive generations.

As will be evident from Section 4, the proposed method differs significantly from the works described in the last couple of paragraphs. It draws inspiration from the neighborhood topologies used in PSO [37]. Similar to DE, PSO has also emerged as a powerful real parameter optimization technique during the late 1990s. It emulates the swarm behavior of insects, animals herding, birds flocking, and fish schooling, where these swarms search for food in a collaborative manner. A number of significantly improved variants of basic PSO have been proposed in recent past to solve both benchmark and real-world optimization problems, for example, see [38, 39]. Earlier attempts to hybridize DE with different operators of the PSO algorithm may be traced to the works of Zhang *et al.* [40] and Das *et al.* [41].

4 DE with a Neighborhood-based Mutation Operator

4.1 The DE/target-to-best/1 --- A few Drawbacks

Most of the population-based search algorithms try to balance between two contradictory aspects of their performance: *exploration* and *exploitation*. The first one means the ability of the algorithm to 'explore' or search every region of the feasible search space while the second denotes the ability to converge to the near-optimal solutions as quickly as possible. The DE variant known as DE/target-to-best/1 (equation (5)) uses the best vector of the population to generate donor vectors. By 'best' we mean the vector that corresponds to the best fitness (e.g., the lowest objective function value for a minimization problem) in the entire population at a particular generation. The scheme promotes exploitation since all the vectors/genomes are attracted towards the same best position (pointed to by the 'best' vector) on the fitness landscape through iterations, thereby converging faster to that point. But as a result of such exploitative tendency, in many cases, the population may lose its global exploration abilities within a relatively small number of generations, thereafter getting trapped to some locally optimal point in the search space.

In addition, DE employs a greedy selection strategy (the better between the target and the trial vectors is selected) and uses a fixed scale factor *F* (typically in [0.4, 1]). Thus if the difference vector $\vec{X}_{r_1,G} - \vec{X}_{r_2,G}$, used for perturbation is small (this is usually the case when the vectors come very close to each other and the population converges to a small domain), the vectors may not be able to explore any better region of the search space, thereby finding it difficult to escape large plateaus or suboptimal peaks/valleys. Mezura-Montes *et al.,* while comparing the different variants of DE for global optimization in [16], have noted that DE/target-to-best/1 shows a poor performance and remains inefficient in exploring the search space, especially for multi-modal functions. The same conclusions were reached by Price *et al.* [3, page 156].

4.2 Motivations for the Neighborhood-based Mutation

A proper trade–off between exploration and exploitation is necessary for the efficient and effective operation of a population-based stochastic search technique like DE, PSO etc. The DE/target-to-best/1, in its present form, favors exploitation only, since all the vectors are attracted by the same best position found so far by the entire population, thereby converging faster towards the same point.

In this context we propose two kinds of neighborhood models for DE. The first one is called the *local neighborhood model*, where each vector is mutated using the best position found so far in a small neighborhood of it and *not* in the entire population. On the other hand, the second one, referred to as the *global mutation model*, takes into account the globally best vector $\vec{X}_{best,G}$ of the entire population at current generation G for mutating a population member. Note that DE/target-to-best/1 employs only the global mutation strategy.

A vector's neighborhood is the set of other parameter vectors that it is connected to; it considers their experience when updating its position. The graph of inter-connections is called the neighborhood structure. Generally, neighborhood connections are independent of the positions pointed to by the vectors. In the *local* model, whenever a parameter vector points to a good region of the search space, it only directly influences its immediate neighbors. Its second degree neighbors will only be influenced after those directly connected to them become highly successful themselves. Thus, there is a delay in the information spread through the population regarding the best position of each neighborhood. Therefore, the attraction to specific points is weaker, which prevents the population from getting trapped in local minima. We would like to mention here that vectors belonging to a *local neighborhood* are not necessarily local in the sense of their geographical nearness or similar fitness values. As will be seen in the next section, the overlapping neighborhoods have been created in DE according to the order of the indices of the population members, following the neighborhood models in PSO.

Finally we combine the local and the global model using a *weight facto*r that appears as a new parameter in the algorithm. The weight factor may be tuned in many different ways. In what follows we describe these issues in sufficient details. Note that the neighborhoods of different vectors were chosen randomly and not according to their fitness values or geographical locations on the fitness landscape, following the PSO philosophy [37]. This preserves the diversity of the vectors belonging to the same neighborhood.

4. 3 The Local and Global Neighborhood-based mutations in DE

Suppose we have a DE population $P_G = [\vec{X}_{1,G}, \vec{X}_{2,G}, ..., \vec{X}_{NP,G}]$ where each $\vec{X}_{i,G}$ $(i = 1,2,..., NP)$ is a D-dimensional parameter vector. The vector indices are sorted only randomly (as obtained during initialization) in order to preserve the diversity of each neighborhood. Now for every vector $\vec{X}_{i,G}$ we define a neighborhood of radius k (where k is a non-zero integer from 0 to $(NP-1)/2$, as the neighborhood size must be smaller than the population size, i.e. $2k+1 \le NP$), consisting of vectors $\vec{X}_{i-k,G}, ..., \vec{X}_{i,G}, ..., \vec{X}_{i+k,G}$. We assume the vectors to be organized on a ring topology with respect to their indices, such that vectors $\vec{X}_{NP,G}$ and $\vec{X}_{2,G}$ are the two immediate neighbors of vector $\vec{X}_{1,G}$. The concept of local neighborhood is schematically illustrated in Figure 2. Note that the neighborhood topology is static and has been defined on the set of indices of the vectors. Although various neighborhood topologies (like star, wheel, pyramid, 4-clusters, and circular) have been proposed in the literature for the PSO algorithms [42], after some initial experimentation over numerical benchmarks, we find that in the case of DE (where the population size is usually larger than in the case of PSO) the circular or ring topology provides best performance compared to other salient neighborhood structures.

For each member of the population a local donor vector is created by employing the best (fittest) vector in the neighborhood of that member and any two other vectors chosen from the same neighborhood. The model may be expressed as:

$$\vec{L}_{i,G} = \vec{X}_{i,G} + \alpha \cdot (\vec{X}_{n\_best_i,G} - \vec{X}_{i,G}) + \beta \cdot (\vec{X}_{p,G} - \vec{X}_{q,G}), \tag{14}$$

where the subscript *n_best_i* indicates the best vector in the neighborhood of $\vec{X}_{i,G}$ and $p, q \in [i-k, i+k]$ with $p \neq q \neq i$. Similarly the global donor vector is created as:

$$\vec{g}_{i,G} = \vec{X}_{i,G} + \alpha \cdot (\vec{X}_{g\_best,G} - \vec{X}_{i,G}) + \beta \cdot (\vec{X}_{r_1,G} - \vec{X}_{r_2,G}), \tag{15}$$

where the subscript *g_best* indicates the best vector in the entire population at generation *G* and $r_1, r_2 \in [1, NP]$ with $r_1 \neq r_2 \neq i$. α and β are the scaling factors.

Note that in equations (14) and (15), the first perturbation term on the right hand side (the one multiplied by α) is an arithmetical recombination operation, while the second term (the one multiplied by β) is the differential mutation. Thus in both the global and local mutation models, we basically generate mutated recombinants, not pure mutants.

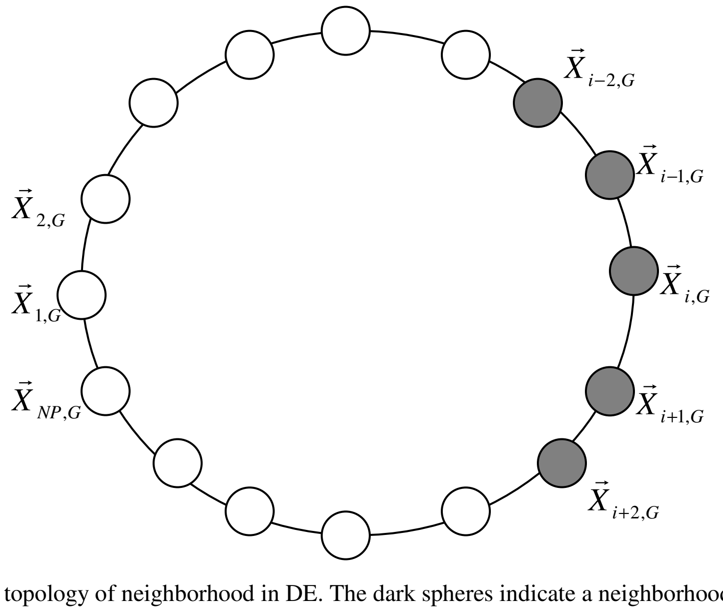


Fig. 2. The ring topology of neighborhood in DE. The dark spheres indicate a neighborhood of radius 2 of the *i*-th population member where *i* = 9.

Now we combine the local and global donor vectors using a scalar weight $w \in (0,1)$ to form the actual donor vector of the proposed algorithm:

$$\vec{V}_{i,G} = w.\vec{g}_{i,G} + (1-w).\vec{L}_{i,G}. \tag{16}$$

Clearly, if *w* = 1and in addition $\alpha = \beta = F$, the donor vector generation scheme in (16) reduces to that of DE/target-to-best/1. Hence the latter may be considered as a special case of this more general strategy involving both global and local neighborhood of each vector synergistically. From now on, we shall refer to this version as DEGL (DE with Global and Local neighborhoods). The rest of the algorithm is exactly similar to DE/rand/1/bin. DEGL uses a binomial crossover scheme and follows the pseudo-code given in Section 3.

Note that in each generation, the vectors belonging to a DE population are perturbed sequentially. If a target vector $\vec{X}_{i,G}$ is replaced with the corresponding trial vector $\vec{U}_{i,G}$, the neighborhood-best $\vec{X}_{n\_best_i,G}$ and the globally best vector $\vec{X}_{g\_best,G}$ may also be updated by $\vec{U}_{i,G}$, provided the latter yields a lower value of the objective function. In Section 4.5, we discuss the additional computational complexity of updating the neighborhood-best vectors in DEGL after the replacement of each target vector in a generation.

4. 4 Control Parameters in DEGL

DEGL introduces four new parameters: α, β, w, and the neighborhood radius *k*. Among them α and β are playing the same role as the constant *F* in (5). Thus, in order to reduce the number of parameters further, we take $\alpha = \beta = F$. The most crucial parameter in DEGL is perhaps the weight factor *w*, which controls the balance between the exploration and exploitation capabilities. Small values of *w* (close to 0) in (16) favor the local neighborhood component, thereby resulting in better exploration. On the other hand, large values (close to 1) favor the global variant component, promoting exploitation. Therefore, values of *w* around the middle point, 0.5, of the range [0, 1] result in the most balanced DEGL versions. However, such balanced versions do not take full advantage of any special structure of the problem at hand (e.g., uni-modality, convexity etc.). In such cases, weight factors that are biased towards 0 or 1 may exhibit better performance. Moreover, on-line adaptation of *w* during the execution of the algorithm can enhance its performance. Optimal values of the weight factor will always depend on the problem at hand. We considered three different schemes for the selection and adaptation of *w* to gain intuition regarding DEGL's performance and we describe them in the following paragraphs.

1) Increasing Weight Factor: All vectors have the same weight factor which is initialized to 0 and is increased up to 1 during the execution of the algorithm. Thus, exploration is favored in the first stages of the algorithm's execution (since *w* = 0 corresponds to the local neighborhood model) and exploitation is promoted at the final stages, when *w* assumes higher values. Let *G* denote the generation number, w_G the weight factor at generation *G*, and G_{max} the maximum number of generations. We considered two different increasing schedules in our study:

A) Linear Increment: *w* is linearly increased from 0 to 1:

$$w_G = \frac{G}{G_{\max}} \cdot \tag{17}$$

B) Exponential Increment: The weight factor increases from 0 to 1 in an exponential fashion as follows:

$$w_G = \exp\left(\frac{G}{G_{\max}} . \ln(2)\right) - 1 \cdot \tag{18}$$

This scheme results in slow transition from exploration to exploitation in the early stages of the algorithm's execution, but exhibits faster transition in the later stages.

2) Random Weight Factor: In this scheme the weight factor of each vector is made to vary as a uniformly distributed random number in (0, 1) i. e. $w_{i,G} \sim rand(0,1)$. Such a choice may decrease the convergence speed (by introducing more diversity).

3) Self Adaptive Weight Factor: In this scheme, each vector has its own weight factor. The factor is incorporated in the vector as an additional variable, augmenting the dimension of the problem. Thus, a generation now consists of vectors $\vec{a}_{i,G} = \{\vec{X}_{i,G}, \vec{S}_{i,G}\}$ where $\vec{S}_{i,G} = \{w_{i,G}\}$ and $w_{i,G}$ is the weight factor for vector $\vec{X}_{i,G}$. During the initialization phase of DE, $w_{i,G}$ is randomly initialized in (0.0, 1.0). Next, while evolving a vector $\vec{a}_{i,G}$, at first local and global mutant vectors $\vec{L}_{i,G}$ and $\vec{g}_{i,G}$ are formed for $\vec{X}_{i,G}$ following equations (14) and (15). The sub-vector $\vec{S}$ undergoes global mutation only and weight factors perturbing $\vec{S}$ come from the same population members $\vec{a}_{r_1,G}$ and $\vec{a}_{r_2,G}$, which were also used to form $\vec{g}_{i,G}$. The mutation of $w_{i,G}$ leads to the formation of a new trial weight factor $w^{/}_{i,G}$ according to the following equation:

$$w^{/}_{i,G} = w_{i,G} + F.(w_{g\_best,G} - w_{i,G}) + F.(w_{r_1,G} - w_{r_2,G}), \tag{19}$$

where, $w_{g\_best,G}$ is the weight factor associated with the best parameter vector $\vec{X}_{g\_best,G}$. The value of the newly formed $w^{/}_{i,G}$ is restricted to the range [0.05, 0.95] in the following way:

$$\begin{aligned} &\text{if } w^{/}_{i,G} > 0.95,\ w^{/}_{i,G} = 0.95; \\ &\text{else if } w^{/}_{i,G} < 0.05,\ w^{/}_{i,G} = 0.05, \end{aligned} \tag{20}$$

$w^{/}_{i,G}$ is then used to combine $\vec{L}_{i,G}$ and $\vec{g}_{i,G}$ according to equation (16) and this leads to the formation of the new donor parameter vector $\vec{V}_{i,G}$. The donor vector thus formed exchanges its components with $\vec{X}_{i,G}$ following the binomial crossover and results in the production of the trial vector $\vec{U}_{i,G}$. Note that the weight factor does not undergo crossover. Now, the newly formed weight factor is promoted to the next generation only if $\vec{U}_{i,G}$ yields an equal or lower objective function value as compared to $\vec{X}_{i,G}$, i. e.,

$$\begin{aligned} &\vec{a}_{i,G+1} = \{\vec{X}_{i,G+1} = \vec{U}_{i,G}, \vec{S}_{i,G+1} = \{w^{/}_{i,G}\}\},\ \text{if } f(\vec{U}_{i,G}) \le f(\vec{X}_{i,G}) \\ &\vec{a}_{i,G+1} = \{\vec{X}_{i,G+1} = \vec{X}_{i,G}, \vec{S}_{i,G+1} = \{w_{i,G}\}\},\ \text{otherwise.} \end{aligned} \tag{21}$$

The process is repeated sequentially for each vector in a generation. Note that the weight factors associated with the neighborhood-best and globally best vectors are not updated every time a trial vector replaces the corresponding target. The weight factor for a parameter vector is changed only once according to equations (19) and (20) in each generation. According to the self-adaptation scheme, the dynamics of DEGL are allowed to determine the optimal $w_{i,G}$ for each vector, individually, capturing any special structure of the problem at hand.

Finally we would like to point out that a proper selection of the neighborhood size affects the trade–off between exploration and exploitation. However, there are no general rules regarding the selection of neighborhood size, and it is usually based on the experience of the user. The effect of neighborhood size on the performance of DEGL has been further investigated in Section 6.5.

4. 5 Runtime Complexity of DEGL – a Discussion

Runtime-complexity analysis of the population-based stochastic search techniques like DE, GA etc. is a critical issue by its own right. Following the works of Zielinski *et al.* [43] we note that the average runtime of a standard DE algorithm usually depends on its stopping criterion. While computing the run-time complexity, we usually take into account the fundamental floating-point arithmetic and logical operations performed by an algorithm [44]. We may neglect very simple operations like copy/assignment etc. as these are merely data-transfer operations between the ALU and/or CPU registers and hardly require any complex digital circuitry like adder, comparator etc. [44, 45]. Now, in each generation of DE, a loop over *NP* is conducted, containing a loop over *D*. Since the mutation and crossover operations are performed at the component level for each DE vector, the number of fundamental operations in DE/rand/1/bin is proportional to the total number of loops conducted until the termination of the algorithm. Thus, if the algorithm is stopped after a fixed number of generations G_{max}, then the runtime complexity is $O(NP \cdot D \cdot G_{\max})$.

For DE/target-to-best/1, runtime complexity of finding the globally best vector depends only on comparing the objective function value against the single best vector's objective function value. Note that the best objective function evaluation value must be upgraded for each newly generated trial vector, if it replaces the target vector. Now that means in the worst possible case (when the target vector is always replaced by the trial vector), this is done $NP \cdot G_{\max}$ times. Thus, the overall runtime remains $O(\max(NP \cdot G_{\max}, NP \cdot D \cdot G_{\max})) = O(NP \cdot D \cdot G_{\max})$.

In DEGL, besides the globally best vector, we have to take into account the best vector of each neighborhood as well. Each individual vector is endowed with a small memory, which can keep track of the best vector in its neighborhood and the corresponding objective function value. At the very onset, once all the vectors are initialized, a search is performed to detect the neighborhood-best for each individual. Note that this search is performed only once at $G = 0$. In subsequent generations, these locally best vectors only need to be updated in the memory of the neighboring vectors. This is just like the updating phase of the globally best vector in DE/target-to-best/1 according to step 2.3 of the DE pseudo-code provided earlier. Now let us try to estimate the cost of the initial search. Note that the neighborhoods in DEGL are actually overlapping in nature (on the index-graph) and this is illustrated in Figure 3. Any two adjacent vectors (with respect to their indices) will have $2k + 1 + 1 - 2 = 2k$ number of common neighbors.

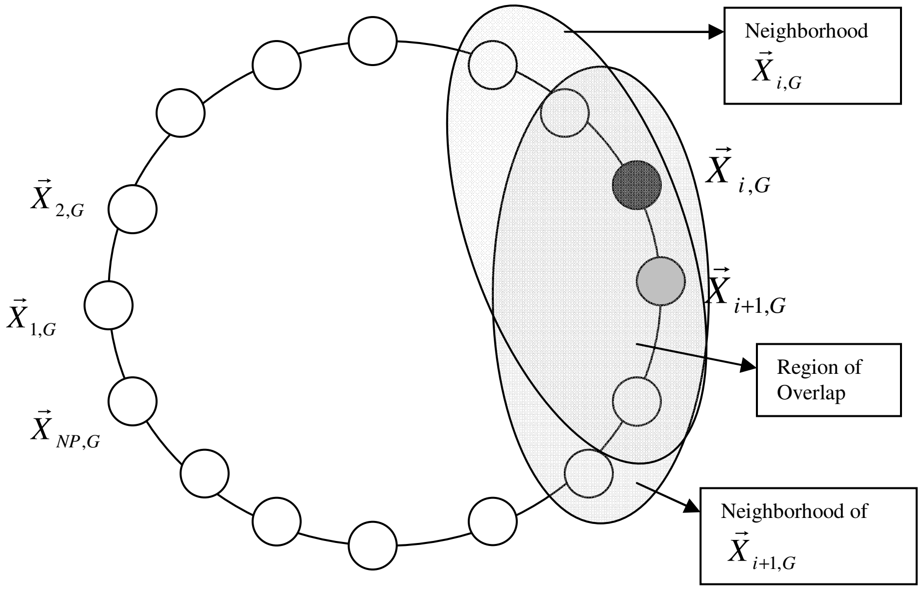


Fig. 3. The overlapping of neighborhoods in DEGL.

Suppose $N_k(\vec{X}_{i,G})$ indicates the set of vectors belonging to the immediate neighborhood of radius k for the vector $\vec{X}_{i,G}$. Then evidently the cardinality of both the sets $N_k(\vec{X}_{i,G}) \cap N_k^c(\vec{X}_{i+1,G})$ and $N_k^c(\vec{X}_{i,G}) \cap N_k(\vec{X}_{i+1,G})$ is exactly 1 (where N_k^c stands for complement of the set N_k). We observe that $\vec{X}_{i-k,G} \in N_k(\vec{X}_{i,G}) \cap N_k^c(\vec{X}_{i+1,G})$ and $\vec{X}_{i+k+1,G} \in N_k^c(\vec{X}_{i,G}) \cap N_k(\vec{X}_{i+1,G})$. Now we start by detecting the best vector of the neighborhood of any population member, say $\vec{X}_{i,G}$ and call it $\vec{X}_{n\_best_i,G}$. This is equivalent to finding the lowest entry from an array of $2k+1$ numbers (objective function values) and requires $2k$ number of comparisons. Next, to calculate the best vector in the neighborhood of $\vec{X}_{i+1,G}$, if $\vec{X}_{n\_best_i,G} \neq \vec{X}_{i-k,G}$ then we simply need to compare the objective function values of $\vec{X}_{i+k+1,G}$ and $\vec{X}_{n\_best_i,G}$ in order to determine $\vec{X}_{n\_best_{i+1},G}$. This requires only one comparison. But if unfortunately $\vec{X}_{n\_best_i,G} = \vec{X}_{i-k,G}$, we shall have to find the neighborhood best of $\vec{X}_{i+1,G}$ by taking its $2k$ neighbors into account and this requires $O(k)$ runtime. Hence in the worst possible case (when the current neighborhood's best vector is always excluded from the serially next vector's neighborhood) searching the best vectors of all the neighborhoods is completed in $O(NP \cdot k)$ time.

Once the search for all neighborhood-bests is finished, in subsequent generations, the best vector in the neighborhood of $\vec{X}_{i,G}$ is updated only if a newly generated trial vector $\vec{U}_{i,G}$ replaces the target vector $\vec{X}_{i,G}$ and in addition to that $f(\vec{U}_{i,G}) < f(\vec{X}_{n\_best_i,G})$. It is possible that $\vec{X}_{n\_best_i,G}$ differs from $\vec{X}_{n\_best_{i+1},G}$ i.e. two vectors, adjacent on the index graph, may have distinct neighborhood-best vectors. This happens when the best vector in the neighborhood of $\vec{X}_{i+1,G}$ is $\vec{X}_{i+k+1,G}$. Under this condition, it is possible that $\vec{U}_{i,G}$ is better than $\vec{X}_{n\_best_i,G}$ but not better than $\vec{X}_{n\_best_{i+1},G}$. Hence in order to update the best vectors in the memories of all the neighbors of $\vec{X}_{i,G}$ (when $f(\vec{U}_{i,G}) < f(\vec{X}_{n\_best_i,G})$ is satisfied) we have to compare the objective function values of $\vec{U}_{i,G}$ and the neighborhood-bests in the memories of $2k$ neighbors of $\vec{X}_{i,G}$. Thus in the worst possible case, updating of all the local best vectors in the memories of the neighbors of each vector requires $O(NP \cdot k)$ comparisons in each generation. Evidently, over $G_{\max}$ generations, the number of additional comparisons necessary is $O(NP \cdot k \cdot G_{\max})$. This implies that the worst case complexity of DEGL is actually $O(\max(NP \cdot k \cdot G_{\max}, NP \cdot D \cdot G_{\max}))$. Now, the asymptotic order of complexity for DEGL remains $O(NP \cdot D \cdot G_{\max})$ if $k \leq D$. Please note that this condition is usually satisfied when DEGL is applied to the optimization of higher dimensional functions. For example, the usual population size for DE is $NP = 10D$. If the neighborhood size is approximately 10% of the population size (which, as can be seen later, provides reasonably good results with DEGL), we have $2k + 1 = (0.1) \cdot NP = D \Rightarrow k = \dfrac{D-1}{2}$ with $D > 1$. Clearly, in this case we have $k \leq D$. Simple algebraic calculations show that this condition holds true if the neighborhood size is below 20% of the population size NP and $D > 1$. Hence, we can say that under such conditions, $O(\max(NP \cdot k \cdot G_{\max}, NP \cdot D \cdot G_{\max})) = O(NP \cdot D \cdot G_{\max})$ and thus DEGL does not impose any serious burden on the runtime complexity of the existing DE variants.

In order to validate the arguments made above, we provide in Table 1 the results of code-function profiling for our implementations of classical DE (DE/rand/1/bin) and DEGL (with random weight factor) using the profiler available with MS Visual C++ 6.0. Both the algorithms were coded in the C language and run on the simple 50-dimensional sphere function (f_1 in the list of benchmarks provided in Table 4). The least complex sphere function was chosen so that most of the CPU time may be spent on the DE operators and not on function evaluations. Here our primary objective is to observe what percentage of the total CPU time is used by the evolutionary operators of DEGL and DE/rand/1/bin. Both algorithms use the same prime modules or code-functions: init_pop (for initializing population), mutate_vector (for performing mutation and creating donor vector), recombine (to perform crossover and create the trial vector), select_and_update (to compare the objective function values of trial and target vectors and in DEGL also to update the neighborhood bests if for the *i*-th vector, the condition $f(\vec{U}_{i,G}) < f(\vec{X}_{n\_best_i,G})$ holds), DE_Operator (module that calls the functions mutate_vector, recombine, and select_and_update for each vector sequentially), evaluate_cost (function that evaluates the objective function for a parameter vector), and the main. The programs were run on a Pentium IV, 2.2 GHz PC, with 512 KB cache and 2 GB of main memory in Windows Server 2003 environment. In Table 1 we provide the code function profiling results as means (with standard deviations in parentheses) of 1000 runs of the programs, each run continued up to 10^5 cost function evaluations (FEs).

Table 1. Code-function runtime profiles for DE/rand/1/bin and DEGL

Algorithm	Total execution time (in milliseconds)	Code-function runtime as % of CPU time						
		init_pop	mutate_ vector	recombine	select_and _update	DE_operator	evaluate_ cost	main
DE/rand/ 1/bin	9382.703 (1825.335)	0.122 (0.0051)	16.728 (0.628)	29.661 (1.481)	8.726 (7.335)	28.824 (3.561)	13.721 (2.727)	2.018 (0.114)
DEGL	9739.684 (1473.627)	0.109 (0.0046)	15.431 (0.937)	16.362 (2.771)	16.839 (6.361)	36.836 (1.663)	12.954 (1.638)	1.469 (0.118)

Table 1 shows that, as expected, the total execution time for DEGL is only marginally higher than that for DE/rand/1/bin. This is because around 16.9% of the total CPU time is consumed by the select_and_update function in DEGL, due to the extra comparisons required for updating the neighborhood-bests. However, if we select a stopping criterion based on a threshold objective function value, instead of the stopping criterion based on maximum number of FEs, DEGL can even take less computation time as compared to DE/rand/1/bin in some cases. This is because DEGL can attain the threshold objective function value much quicker, consuming significantly smaller number of FEs, due to the better trade-off between exploration and exploitation abilities achieved by its neighborhood-based mutation operators. This fact has been illustrated by providing, in Tables 2 and 3, the mean processor-time taken by both the algorithms for both stopping criteria over five most popular benchmark functions used for testing the evolutionary algorithms. Note that both the algorithms start from the same initial population and run under the same software and hardware platforms. All the numerical benchmarks dealt in here are in 25 dimensions, have their true optima at 0.00, and for all of them the target threshold value was set at 1.00e-05 in Tales 2 and 3. A detailed description of these functions can be found in Table 4 in the following section. Each result is the average of 50 independent runs.

We would like to point out that, in the evolutionary computing literature, comparison of the computational costs of various evolutionary algorithms is usually performed on the basis of the number of FEs they take to reach a predefined function-value. Processor time cannot serve as a reliable metric in this context, because first, it is not independent of the hardware and software platforms used and second, it may provide some unfair advantage to algorithms that use lower computational overheads. In addition, the processor time depends on the style of coding an algorithm [46]. The advantage of measuring the runtime complexity by counting the number of FEs is that the correspondence between this measure and the processor time becomes stronger as the function complexity increases. In Section 6, we compare the computational cost and convergence speed of a number of DE-variants using this measure. The tables included in this section are intended only to provide an approximate feel of the relative time-complexities of DEGL and classical DE.

Table 2 shows that when DEGL and DE/rand/1/bin are run for the same number of FEs (corresponding to the same number of generations for both, as they have the same population size), the processor time required by the former is slightly higher than that of the latter. Table 3, however, indicates that DEGL may reach the predefined threshold value with less processor time as compared to DE/rand/1/bin.

Table 2. A comparison of absolute run-times of DEGL and DE/rand/1/bin, when both the algorithms were run for a fixed number of FEs.

Function	Mean processor time (in milliseconds) and standard deviation (in parentheses)	
	DE/rand/1/bin	DEGL
Step function (f_6)	3692.84 (688.25)	3973.38 (827.51)
Rosenbrock's function (f_5)	6726.57 (1425.53)	7061.48 (1930.51)
Rastrigin's function (f_9)	5883.54 (629.63)	6273.38 (447.23)
Ackley's function (f_{11})	5094.68 (1624.83)	5268.46 (324.68)
Griewank's function (f_{12})	5635.92 (1023.35)	6163.28 (729.46)

Table 3. A comparison of absolute run-times of DEGL and DE/rand/1/bin, when both the algorithms were run until they attain a pre-defined objective function value.

Function	Threshold objective-function value to reach	Mean processor time (in milliseconds) and standard deviation (in parentheses)	
		DE/rand/1/bin	DEGL
Step function (f_6)	1.00-05	3022.84 (271.22)	2873.38 (712.58)
Rosenbrock's function (f_5)	1.00-05	5718.92 (1425.53)	5448.37 (1628.31)
Rastrigin's function (f_9)	1.00-05	2483.56 (442.67)	1682.94 (538.19)
Ackley's function (f_{11})	1.00-05	839.68 (154.41)	692.70 (32.61)
Griewank's function (f_{12})	1.00-05	4836.29 (1023.35)	4667.25 (1416.47)

5. Experimental Set-up

5.1 Benchmark Functions

We have used a test-bed of twenty-one traditional numerical benchmarks (Table 4) [47] and three composition functions from the benchmark problems suggested in CEC 2005 [48] to evaluate the performance of the new DE-variant. The 21 traditional benchmarks described by Yao *et al.* have been reported in Table 4 where D represents the number of dimensions. For $f_1 - f_{13}$ we have tested for D = 25 to 100 in steps of 25. Among these benchmarks, functions $f_1 - f_{13}$ are multi-dimensional problems. Functions $f_1 - f_5$ are uni-modal (there is some recent evidence [49] that f_5 is multi-modal for $D > 3$). Function f_6 is a step function with one minimum and is discontinuous. Function f_7 is a noisy quartic function, where *random* [0, 1) is a uniformly distributed random number in [0, 1).

Functions $f_8 - f_{13}$ are multi-modal with the number of local minima increasing exponentially with the problem dimension [47]. They apparently belong to the most difficult class of problems for many optimization algorithms. Functions $f_{14} - f_{21}$ are low-dimensional functions which have only a few local minima. For uni-modal functions, the convergence rates of the DE algorithms are more interesting than the final results of optimization as there are other methods which are specifically designed to optimize uni-modal functions. For multi-modal functions, the final results are much more important since they reflect an algorithm's ability of escaping from poor local optima and locating a good near-global optimum. We omitted f_{19} and f_{20} from Yao *et al.*'s study [47] because of difficulties in obtaining the definitions of the constants used in these functions.

The three composition functions $f_{18}(\vec{X})$, $f_{19}(\vec{X})$, and $f_{21}(\vec{X})$, taken from CEC 2005 benchmarking problems [48], are here marked as CF1, CF2, and CF3 respectively. All of them are non-separable, rotated, and multi-modal functions containing a large number of local optima. For all of them the search range is $\vec{X} \in [-5,5]^D$. The global optimum of both CF1 and CF2 is $f(\vec{X}^*) = 10$ and that for CF3 is $f(\vec{X}^*) = 360$. The detailed principle of the composite functions is given in [48].

For the generalized penalized functions f_{12} and f_{13}, in Table 1, note that

$$\begin{aligned} u(x_i,a,k,m) &= k(x_i - a)^m, && \text{if } x_i > a \\ &= 0, && \text{if } -a \le x_i \le a \\ &= k(-x_i - a)^m, && \text{if } x_i < -a \end{aligned}$$

and $$y_i = 1 + \frac{1}{4}(1 + x_i)\ .$$

Values of the other constants used in the expressions of the benchmark functions can be found in [47].

5.2 Other Optimization Problems Considered

In this section we describe two interesting real-world problems that have been used to test the efficacy of the DEGL family. The problems are selected according to the level of difficulty that they present to the proposed algorithms.

5.2.1 The Spread Spectrum Radar Poly-phase Code Design Problem

A famous problem of optimal design arises in the field of spread spectrum radar poly-phase codes [50]. Such a problem is very well-suited for the application of global optimization algorithms like DE. The problem can be formally stated as:

$$\text{Global min } f(\vec{X}) = \max\{\varphi_1(\vec{X}),....,\varphi_{2m}(\vec{X})\}, \tag{22}$$

$$\text{where } \vec{X} = \{(x_1,....,x_D) \in \Re^D \mid 0 \le x_j \le 2\pi, j = 1,...,D\} \text{ and } m = 2D-1,$$

with
$$\begin{aligned} \varphi_{2i-1}(\vec{X}) &= \sum_{j=i}^{D} \cos\Big(\sum_{k=|2i-j-1|-1}^{j} x_k\Big), && i = 1,2,...,D \\ \varphi_{2i}(\vec{X}) &= 0.5 + \sum_{j=i+1}^{D} \cos\Big(\sum_{k=|2i-j-1|-1}^{j} x_k\Big), && i = 1,2,...,D-1 \\ \varphi_{m+i}(\vec{X}) &= -\varphi_i(\vec{X}), && i = 1,2,...,m. \end{aligned} \tag{23}$$

According to [50] the above problem has no polynomial time solution. The objective function for D = 2 is shown in Figure 4.

Table 4: 21 Traditional Benchmark Functions [47]

Function	D	Search Range	Optimum Value
$f_1(\vec{X})=\sum_{i=1}^{D}x_i^2$	25, 50, 75, and 100	$-100 \le x_i \le 100$	$f_1(\vec{0})=0$
$f_2(\vec{X})=\sum_{i=1}^{D}\lvert x_i\rvert+\prod_{i=1}^{D}x_i$	25, 50, 75, and 100	$-10 \le x_i \le 10$	$f_2(\vec{0})=0$
$f_3(\vec{X})=\sum_{i=1}^{D}(\sum_{j=1}^{i}x_j)^2$	25, 50, 75, and 100	$-100 \le x_i \le 100$	$f_3(\vec{0})=0$
$f_4(\vec{X})=\max \lvert x_i \rvert, 1 \le i \le D$	25, 50, 75, and 100	$-100 \le x_i \le 100$	$f_4(\vec{0})=0$
$f_5(\vec{X})=\sum_{i=1}^{D-1}[100(x_{i+1}-x_i{}^2)^2+(x_i-1)^2]$	25, 50, 75, and 100	$-30 \le x_i \le 30$	$f_5(\vec{1})=0$
$f_6(\vec{X})=\sum_{i=1}^{D}(\lfloor x_i+0.5\rfloor)^2$	25, 50, 75, and 100	$-100 \le x_i \le 100$	$f_6(\vec{p})=0, -\frac{1}{2} \le p_i < \frac{1}{2}$
$f_7(\vec{x})=(\sum_{i=1}^{D}i.x_i^4)+rand[0,1)$	25, 50, 75, and 100	$-1.28 \le x_i \le 1.28$	$f_7(\vec{0})=0$
$f_8(\vec{X})=\sum_{i=1}^{D}-x_i.\sin(\sqrt{\lvert x_i \rvert})$	25, 50, 75, and 100	$-500 \le x_i \le 500$	$f_8(420.97)=-41898.3$ for D = 100
$f_9(x)=\sum_{i=1}^{D}[x_i^2-10\cos(2\pi x_i)+10]$	25, 50, 75, and 100	$-5.12 \le x_i \le 5.12$	$f_9(\vec{0})=0$
$f_{10}(\vec{X})=-20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D}x_i^2}\right)-\exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos 2\pi x_i\right)+20+e$	25, 50, 75, and 100	$-32 \le x_i \le 32$	$f_{10}(\vec{0})=0$
$f_{11}(\vec{X})=\frac{1}{4000}\sum_{i=1}^{D}x_i^2-\prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}})+1$	25, 50, 75, and 100	$-600 \le x_i \le 600$	$f_{11}(\vec{0})=0$
$f_{12}(\vec{X})=\frac{\pi}{D}\{10\sin^2(\pi y_1)+\sum_{i=1}^{D-1}(x_i-1)^2.[1+10\sin^2(\pi y_{i+1})]+(y_D-1)^2\}+\sum_{i=1}^{D}u(x_i,10,100,4)$	25, 50, 75, and 100	$-50 \le x_i \le 50$	$f_{12}(\overrightarrow{-1})=0$
$f_{13}(\vec{X})=0.1\{\sin^2(3\pi x_1)+\sum_{i=1}^{D-1}(x_i-1)^2.[1+\sin^2(3\pi x_{i+1})]+(x_D-1)\{1+\sin^2(2\pi x_n)\}+\sum_{i=1}^{D}u(x_i,5,100,4)$	25, 50, 75, and 100	$-50 \le x_i \le 50$	$f_{13}(1,...,1,-4.76)$ $=-1.1428$
$f_{14}(\vec{X})=(\frac{1}{500}+\sum_{j=1}^{25}(j+1+\sum_{i=0}^{1}(x_i-a_{ij})^6)^{-1}$	2	$-65.54 \le x_i \le 65.54$	$f_{14}(\overrightarrow{-31.95})$ $=0.998$
$f_{15}(\vec{X})=\sum_{i=0}^{10}(a_i-\frac{x_0(b_i^2+b_ix_1)}{b_i^2+b_ix_2+x_3})^2$	4	$-5 \le x_i \le 5$	$f_{15}(0.1928,0.1908,0.1231,0.1358)$ $=0.0003075$
$f_{16}(\vec{X})=4x_0^2-2.1x_0^4+\frac{1}{3}x_0{}^6+x_0x_1-4x_1{}^2+4x_1{}^4$	2	$-5 \le x_i \le 5$	$f_{16}(-0.09,0.71)$ $=-1.0316$
$f_{17}(\vec{X})=(x_1-\frac{5.1}{4\pi^2}x_0{}^2+\frac{5}{\pi}x_0-6)^2+10(1-\frac{1}{8\pi})\cos(x_0)+10$	2	$-5 \le x_i \le 5$	$f_{17}(9.42,2.47)$ $=0.398$
$f_{18}(\vec{X})=\{1+(x_0+x_1+1)^2(19-14x_0+3x_0{}^2-14x_1-6x_0x_1+3x_1{}^2)\}\{30+(2x_0-3x_1)^2(18-32x_0+12x_0{}^2+48x_1-36x_0x_1+27x_1{}^2)\}$	2	$-2 \le x_i \le 2$	$f_{18}(1.49e-05,1.00)$ $=3$
$f_{19}(\vec{X})=-\sum_{i=1}^{5}((\vec{X}-\vec{a}_i)^T(\vec{X}-\vec{a}_i)+c_i)^{-1}$	4	$-10 \le x_i \le 10$	$f_{19}(\vec{4})=-10.1532$
$f_{20}(\vec{X})=-\sum_{i=1}^{7}((\vec{X}-\vec{a}_i)^T(\vec{X}-\vec{a}_i)+c_i)^{-1}$	4	$-10 \le x_i \le 10$	$f_{20}(\vec{4})=-10.4029$
$f_{21}(\vec{X})=-\sum_{i=1}^{10}((\vec{X}-\vec{a}_i)^T(\vec{X}-\vec{a}_i)+c_i)^{-1}$	4	$-10 \le x_i \le 10$	$f_{21}(\vec{4})=-10.5364$

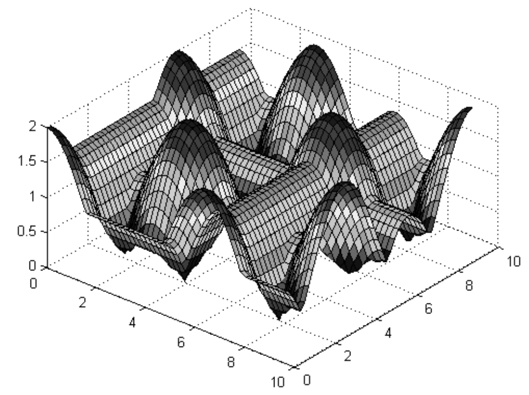


Fig. 4. $f(\vec{X})$ of equation (22) for $D = 2$.

5.2.2 Application to Parameter Estimation for Frequency-Modulated (FM) Sound Waves

Frequency-modulated (FM) sound synthesis plays an important role in several modern music-systems. This section describes an interesting application of the proposed DE algorithms to the optimization of parameters of an FM synthesizer. A few related works that attempt to estimate parameters of the FM synthesizer using the genetic algorithm can be found in [51, 52]. Here we introduce a system that can automatically generate sounds similar to the target sounds. It consists of an FM synthesizer, a DE optimizer, and a feature extractor. The system architecture is shown in Figure 5. The target sound is a *.wav* file. The DE algorithm initializes a set of parameters and the FM synthesizer generates the corresponding sounds. In the feature extraction step, the dissimilarities of features between the target sound and synthesized sound are used to compute the fitness value. The process continues until synthesized sounds become very similar to the target.

The specific instance of the problem discussed here involves determination of six real parameters: $\vec{X} = \{a_1, \omega_1, a_2, \omega_2, a_3, \omega_3\}$ of the FM sound wave given by equation (24) for approximating it to the sound wave given in (25) where $\theta = 2\pi/100$. The parameters are defined in the range [-6.4, +6.35]. The formula for the estimated sound wave and the target sound wave may be given as:

$$y(t) = a_1.\sin(\omega_1.t.\theta + a_2.\sin(\omega_2.t.\theta + a_3.\sin(\omega_3.t.\theta))) \tag{24}$$

$$y_0(t) = 1.0.\sin(5.0.t.\theta - 1.5.\sin(4.8.t.\theta + 2.0.\sin(4.9.t.\theta))) \tag{25}$$

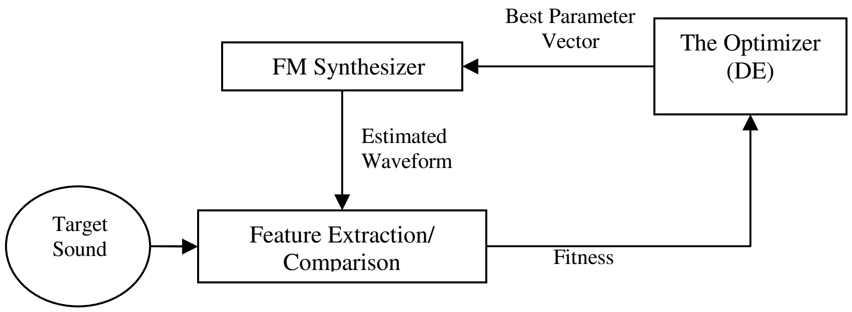


Fig. 5. Architecture of the optimization system.

The goal is to minimize the sum of squared errors between the estimated sound and the target sound, as given by (26). This problem involves a highly complex multi-modal function having strong epistasis (interrelation among the variables), with optimum value 0.0.

$$f(\vec{X}) = \sum_{t=0}^{100} (y(t) - y_0(t))^2 . \tag{26}$$

Owing to the great difficulty of solving this problem with high accuracy without specific operators for continuous optimization (like gradual GAs [52]), we stop the algorithm when the number of function evaluations exceeds 10^5. As in the previous experiments, here also the runs of the competing DE variants start with the same initial population.

5.3 Algorithms for Comparison

At first four versions of the proposed DEGL algorithm (with different schedules for changing the weight factor *w*) are compared with the DE/target-to-best/1/bin. These four versions are referred to as DEGL/LIW (DEGL with Linearly Increasing Weight factor), DEGL/EIW (DEGL with Exponential Increasing Weight factor), DEGL/RandW (DEGL with Random Weight factor) and DEGL/SAW (DEGL with Self Adaptive Weight factor). We included a DEGL algorithm with a fixed value of *w* for all the vectors in this comparative study. For this scheme we choose $w = 0.5$ (which provides equal importance to both local and global mutation schemes and appears to be the best performer as compared to other fixed values of *w* varying between 0.1 to 1.0 in steps of 0.1). The reason for including this scheme is to illustrate the effectiveness of the time-varying or adaptive weight factor over a fixed weight factor. In order to investigate the effect of the explorative mutation operator, the local-only DEGL (with $w = 0$) was also taken into account in the comparative study.

Simulations were carried out to obtain a comparative performance analysis of DEGL/SAW (that appears to be the best performing algorithm from the first set of experiments) with respect to: (a) DE/rand/1/bin [1] (b) DE/target-to-best/1/bin [19] (c) DE/rand/1/either-or [3] (d) SADE [28] and (e) NSDE [31]. Among the competitors, the first two belong to the classical DE family of Storn and Price. The DE/rand/1/bin algorithm was chosen because of its wide popularity in solving numerical optimization or engineering problems [3].

5.4 Initial Population and Method of Initialization

For all the contestant algorithms we used the same population size, which is 10 times the dimension *D* of the problem. To make the comparison fair, the populations for all the DE variants (over all problems tested) were initialized using the same random seeds. Fogel and Beyer [53] have shown that the typical method of symmetric initialization, used to compare evolutionary computations can give false impressions of relative performance. In many comparative experiments, the initial population is uniformly distributed about the entire search space which is usually defined to be symmetric about the origin. In addition, many of the test functions are crafted to have optima at or near the origin, including the test functions for this study. A uniform distribution of initial population members has two potential biases for such functions. In this work we have adopted an asymmetrical initialization procedure following the work reported in [54]. The procedure limits the initial process to just a portion of the feasible search space (as shown in the third column of Table 4), which is a region defined to be half the distance from the maximum point along each axis back toward the origin. Consequently, as the number of dimensions is increased, the volume of the initialization space in the asymmetric initialization procedure decreases exponentially as compared to that of the symmetric initialization (whose limits are provided in Table 4).

For the spread spectrum radar code design problem, each variable is randomly initialized in the interval $[0, 2\pi]$. The search was kept confined in this region. On the other hand, for the FMS problem, the initialization range of each of the six variables was kept at [0, 6.35], while the search was constricted in the region [-6.4, 6.35] for all the variables.

6 Numerical Results and Discussions

6.1 Comparison of Different DEGL Schemes

In this section we compare the performance of six variants of the proposed DEGL algorithm (with different strategies for tuning the weight factor *w*) and the DE/target-to-best/1 scheme, which uses only a global neighborhood and may be seen as a special case of the DEGL with $w = 1$ and $\alpha = \beta$. All the seven contestant algorithms in this section use the same population size, the same intial population, and the same stopping criterion (i.e. the same number of maximum FEs). Here the results are shown for D = 100 and each run of an algorithm is continued upto 5,000,000 FEs. Since all the algorithms have the same population size $(10 \cdot D)$, this corresponds to a maximum of approximately 5000 generations for each problem.

In the self-adaptive scheme (DEGL/SAW) for adjusting w, the weight-factor of each vector was randomly initialized, using a uniform distribution, and constrained within [0.05, 0.95]. This range gave fairly good results with DEGL/SAW algorithm.

We choose the crossover rate Cr = 0.9, scale factors $\alpha = \beta = F = 0.8$. After some experimentation we find that a neighborhood size approximately equal to 10% of the population size provides reasonably accurate results for DEGL over nearly all the problems we study here. Hence we stick to a 10% neighborhood size everywhere in this comparative study for DEGL. Section 6.5 presents a detailed discussion of the effect of the neighborhood size on DEGL's performance.

The mean and the standard deviation (within parentheses) of the best-of-run values for 50 independent runs of each of the five contestant algorithms are presented in Table 5 for the six hardest benchmark functions functions f_8 to f_{13} (each in 100 dimensions) and also for the three composite functions CF1 to CF3 (each in 10 dimensions), taken from the list of CEC'05 benchmarks [48]. The best solution in each case has been shown in bold. Final accuracy results for all the algorithms studied here have been reported with precision as recorded by the IEEE Standard for Binary Floating-Point Arithmetic (IEEE 754). Results for relatively easier benchmarks follow a similar trend and have not been included in order to save space.

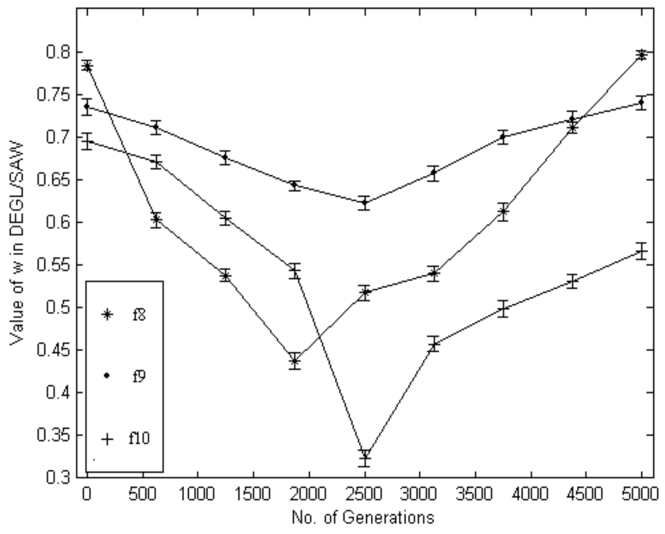
Table 5. Average and the standard deviation of the best-of-run solution for 50 independent runs and the success rate tested on functions f_8 to f_{13} and composite functions CF1 to CF3.

functions	Mean Best Value (Standard Deviation)						
	DE/target-to-best/1/bin	DEGL with fixed $w = 0.5$	DEGL with $w = 0$	DEGL/LI	DEGL/EI	DEGL /RandW	DEGL/SAW ($Cr = 0.9$)
f_8	-3.94382e+04 (5.83e-06)	-3.8756e+04 (7.00e-06)	-3.5621e+04 (8.58e-06)	-4.03634e+04 (3.81e-05)	-4.18436e+04 (5.22e-05)	-4.09039e+04 (8.39e-06)	**-4.18983e+04 (6.98e-06)**
f_9	8.38673e-02 (5.06e-03)	8.35525e-02 (4.96e-02)	5.1215e-03 (3.81e-03)	3.46138e-06 (5.91e-07)	2.90833e-06 (5.91e-06)	8.93821e-21 (5.4342e-18)	**1.7728e-26 (3.88e-25)**
f_{10}	6.76249e-01 (4.27e-01)	6.65735e-01 (7.07e-01)	2.09364e-01 (4.38e-01)	5.48844e-02 (1.68e-01)	3.93270e-04 (3.28e-02)	3.00895e-10 (7.16e-07)	**8.52742e-17 (1.365e-15)**
f_{11}	5.27284e-05 (4.63e-07)	9.07997e-06 (9.02e-05)	6.46925e-06 (3.49e-08)	8.63652e-06 (1.02e-04)	4.82634e-06 (3.63e-06)	8.92369e-12 (6.02e-13)	**4.11464e-15 (6.02e-16)**
f_{12}	5.21919e-02 (2.94e-04)	5.25646e-03 (7.15e-06)	5.25646e-03 (7.15e-06)	4.34325e-04 (3.69e-05)	5.13084e-03 (3.59e-04)	4.74317e-04 (4.05e-05)	**3.00496e-18 (4.82e-17)**
f_{13}	2.30179e+01 (4.38e-01)	1.35424e+01 (3.67e-02)	1.77582e+01 (6.33e-04)	-4.86485e-01 (1.08e-10)	-1.00864e+00 (1.44e-05)	-1.10554e+00 (6.98e-02)	**-1.14282e+00 (9.02e-05)**
CF1	7.35430e+02 (1.546e+02)	7.36630e+02 (4.326e+01)	7.37321e+02 (7.235e+01)	6.98553e+02 (1.236e+02)	6.98661e+02 (2.123e+02)	7.847894e+02 (3.353e+02)	**6.19227e+02 (6.8341e+01)**
CF2	8.65593e+02 (2.541e+02)	8.54723e+02 (2.482e+01)	8.34774e+02 (1.554e+01)	7.82114e+02 (1.231e+02)	6.70442e+02 (1.133e+02)	6.40562e+02 (2.643e+02)	**5.60543e+02 (9.7837e+01)**
CF3	9.73340e+02 (3.221e+02)	9.13774e+02 (5.689e+02)	9.18563e+02 (4.663e+01)	1.12504e+03 (2.236e+02)	8.16728e+02 (2.836e+02)	8.41423e+02 (2.643e+02)	**6.74823e+02 (5.8471e+01)**

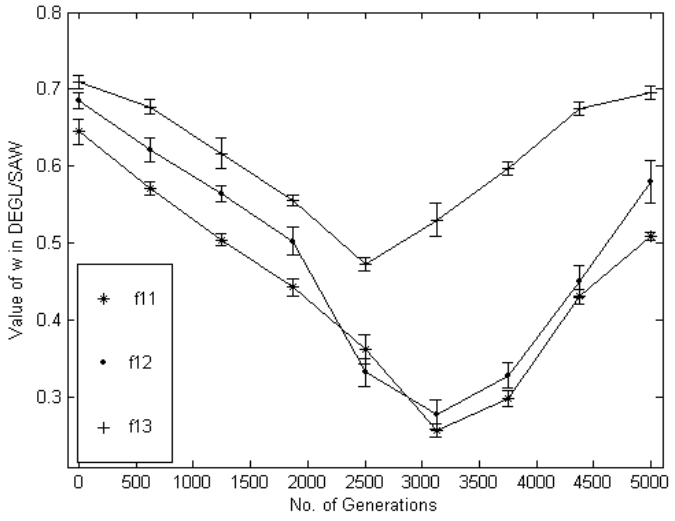
From Table 5, it is interesting to see that there are always one or more versions of DEGL that outperform the standard DE/target-to-best/1/bin scheme. This reflects the effectiveness of the incorporation of the hybrid mutation operator in DE. We also note that in all the cases the time-varying weight-factors outperform the schemes with fixed weight-factor. It is interesting to see that DEGL with a fixed w for all vectors yields final accuracies very close to that produced by the DE/target-to-best/1/bin scheme. However, performance of the *local-only* DEGL with w = 0 remains comparable to DEGL with w = 0.5 but poorer than the three other DEGL schemes with time-varying weight factor. Most of the runs of DEGL with w = 0 fail to converge very near to the global optima within the prescribed number of FEs due to its sluggish behavior during the final stages of the search. This suggests that a judicious trade-off between the explorative and the exploitative mutation operators is the key to the success of the search-dynamics of DEGL. The self-adaptive DEGL/SAW scheme exhibited very good performance over all the test problems, indicating the ability of DEGL to capture the dynamics of the problem under test and determine the proper weight-factor. In Figure 6 the evolution of the weight-factor over successive generations has been shown for the best vector of the median run of DEGL/SAW over functions f_8 - f_{13}. The standard deviations have also been plotted at the sampled generations in the same figure.

Very interestingly, Figure 6 indicates that the general tendency of the evolutionary learning is at first a decrease of the weight factor (favoring exploration at earlier stages) and then increasing the weight factor towards a high value (favoring exploitation at later stages of the search).

In the following sections we report results of comparison between DEGL/SAW and other state-of-the-art DE variants. We exclude the other variants of DEGL to save space and also considering the fact that DEGL/SAW outperformed all other schemes of controlling the weight factor over the selected test-suite.



(a) Variation of w for DEGL/SAW over functions f_8 to f_{10}



(b) Variation of w for DEGL/SAW over functions f_{11} to f_{13}

Fig. 6. Self-adaptation characteristics of the best vector of median run for the DEGL/SAW scheme.

6.2 Comparison of DEGL/SAW with State-of-the-art DE-Variants

In this section, we compare DEGL/SAW with five other DE variants mentioned in Section 5.3. The comparative study focuses on four important aspects of all the competitor algorithms: (a) The quality of the final solutions produced by each algorithm, irrespective of the computational time it consumes, (b) The speed of convergence measured in terms of the number of FEs required by an algorithm to reach a predefined threshold value of the objective function, (c) the frequency of hitting the optima (or *success rate*) measured in terms of the number of runs of an algorithm that converge to a threshold value within a predetermined number of FEs, and (d) the issue of scalability, i.e. how the performance of an algorithm changes with the growth of the search-space dimensionality.

The parametric setup for DEGL was kept same as before. For DE/rand/1/bin and DE/target-to-best/1/bin we have taken $F = 0.8$, $Cr = 0.9$, and $NP = 10 \cdot D$. In the case of DE/rand/1/either-or, we took $p_F = 0.4$ [3]. For NSDE and SADE, the best set of parameters was employed from the relevant literature ([31] and [28] respectively). Once set, the same parameters were used over all the tested problems and no further hand tuning was allowed for any of the algorithms.

6.2.1 Comparison of Quality of the Final Solution

To judge the accuracy of different DE variants, we first let each of them run for a very long time over every benchmark function, until the number of FEs exceeds a given upper limit (which was fixed depending on the complexity of the problem). The mean and the standard deviation (within parentheses) of the best-of-run values for 50 independent runs of each of the six algorithms are presented in Tables 6, 7, and 8. Missing standard deviation values in any result table in this paper indicate zero standard deviation. Although the experiments were conducted for D = 25, 50, 75, and 100 for functions f_1 to f_{13}, we report here results for 25 and 100 dimensions in order to save space. Please note that the omitted results follow a similar trend as those reported in Tables 6, 7, and 8.

Since all the algorithms start with the same initial population over each problem instance, we used paired *t*-tests to compare the means of the results produced by best and the second best algorithms (with respect to their final accuracies). The *t*-tests are quite popular among researchers in evolutionary computing and they are fairly robust to violations of a Gaussian distribution with large number of samples like 50 [55]. In the 10-th columns of Tables 6, 7, and 8 we report the statistical significance level of the difference of the means of best two algorithms. Note that here '+' indicates the *t* value of 49 degrees of freedom is significant at a 0.05 level of significance by two-tailed test, '.' means the difference of means is not statistically significant and 'NA' stands for Not Applicable, covering cases for which two or more algorithms achieve the best accuracy results.

Table 6. Average and the standard deviation of the best-of-run solution for 50 independent runs and the success rate tested on f_1 to f_8

Func tion	Dim	Max FEs	Mean Best Value (Standard Deviation)						
			DE/rand/1 /bin	DE/target-to-best/1/ bin	DE/rand/1/ either-or	SADE [27]	NSDE [30]	DEGL/ SAW	Statistical Significance
f_1	25	5×10^5	6.8594e-29 (4.984e-23)	5.7093e-25 (2.109e-19)	7.3294e-36 (5.394e-34)	4.0398e-35 (3.905e-32)	9.5462e-35 (3.009e-34)	**8.7845e-37 (3.823e-35)**	•
	100	5×10^6	8.4783e-24 (4.664e-22)	2.5693e-23 (3.746e-21)	**4.9382e-26 (4.9382e-25)**	5.8472e-24 (3.8271e-23)	8.3812e-23 (3.925e-25)	3.6712e-25 (4.736e-23)	•
f_2	25	5×10^5	7.5462e-29 (6.731e-29)	5.7362e-25 (4.837e-10	7.4723e-31 (2.736e-34)	8.3392e-26 (4.837e-28)	8.9437e-30 (1.003e-30)	**4.9392e-36 (3.928e-34)**	+
	100	5×10^6	1.6687e-09 (6.77e-10)	3.5273e-06 (1.68e-08)	6.2827e-13 (1.91e-15)	2.6595e-12 (3.36e-14)	9.1395e-10 (3.36e-10)	**6.9982e-14 (1.34e-16)**	+
f_3	25	5×10^5	4.9283e-11 (2.03e-11)	6.2713e-09 (4.82e-10)	5.8463e-24 (4.737e-24)	4.2761e-14 (3.87e-14)	3.0610e-09 (4.22e-10)	**1.2094e-26 (3.827e-25)**	+
	100	5×10^6	6.5712e-10 (2.91e-10)	5.6125e-10 (3.22e-12)	3.4315e-11 (5.07e-12)	4.5641e-10 (5.29e-13)	7.3412e-10 (6.12e-10)	**5.8832e-13 (3.06e-16)**	+
f_4	25	5×10^5	8.3611e-14 (6.37e-13)	5.3711e-10 (9.03e-09)	1.6281e-14 (3.42e-13)	3.0229e-14 (1.37e-15)	2.0936e-11 (1.09e-08)	**4.9932e-15 (1.18e-14)**	+
	100	5×10^6	3.0095e-12 (3.26e-11)	3.0005e-08 (3.69e-09)	9.4442e-13 (3.29e-14)	3.7001e-11 (1.08e-13)	6.0927e-09 (4.45e-08)	**3.5677e-14 (4.55e-13)**	+
f_5	25	5×10^5	9.8372e-23 (4.837e-24)	3.0345e-10 (3.69e-09)	4.9372e-25 (3.726e-21)	**5.6472e-26 (9.367e-24)**	2.6473e-25 (4.536e-25)	6.8948e-25 (4.361e-26)	•
	100	5×10^6	8.4511e-05 (2.748e-05)	2.6183e-01 (1.329e-03)	8.5462e-23 (4.635e-23)	8.6471e-25 (3.782e-24)	5.9208e-08 (2.03e-09)	**1.5463e-25 (7.301e-22)**	•
f_6	25	5×10^5	6.0938e-32 (9.362e-40)	7.6473e-41 (3.827e-37)	2.6839e-45 (3.837e-43)	1.6729e-36 (2.637e-32)	4.0361e-28 (2.949e-34)	**9.5627e-48 (2.732e-45)**	+
	100	5×10^6	3.2387e-14 (2.67e-09)	4.0102e-12 (3.85e-13)	8.3026e-15 (5.51e-16)	6.4897e-21 (3.938e-19)	5.8924e-15 (6.00e-13)	**9.4826e-22 (7.483e-24)**	+
f_7	25	5×10^5	4.9391e-03 (5.92e-04)	9.0982e-03 (2.08e-04)	6.9207e-04 (4.26e-06)	3.7552e-02 (9.02e-03)	4.3482e-03 (6.50e-04)	**1.0549e-07 (2.33e-06)**	+
	100	5×10^6	2.8731e-02 (2.33e-02)	3.3921e-02 (3.32e-02)	4.3332e-03 (5.76e-02)	5.9281e-02 (4.31e-03)	9.8263e-02 (2.90e-03)	**6.9921e-06 (4.56e-05)**	+
f_8	25	5×10^5	-1.0182e+04 (2.83e-04)	-1.0236e+04 (3.81e-05)	**-1.0475e+04 (2.27e-06)**	**-1.0475e+04 (2.27e-06)**	-1.1472e+04 (2.91e-03)	**-1.0475e+04 (3.77e-03)**	NA
	100	5×10^6	-4.18315e+04 (2.83e-04)	-3.9382e+04 (5.83e-06)	-4.18445e+04 (5.22e-05)	-4.18091e+04 (2.49e-06)	-4.18091e+04 (2.49e-06)	**-4.18983e+04 (6.98e-06)**	•

A close inspection of Tables 6 - 8 indicates that the performance of the proposed DEGL/SAW algorithm has remained clearly and consistently superior to that of the two classical DE schemes (DE/rand/1/bin and DE/target-to-best/1/bin) as well as the three state-of-the-art DE variants. One may note from Tables 6 and 7 that for a few relatively simpler test-functions like the Sphere (f_1), Schwefel's problem 2.22 (f_2), 25-dimensional Step function (f_6), generalized Rastrigin's function (f_9), generalized Griewank's function (f_{11}) and the Shekel's family function f_{22}, most of the algorithms end up with almost equal accuracy. Substantial performance differences however, are noticed for the rest of the more challenging benchmark functions and especially for functions with higher dimensions like 100. In the case of the multi-modal functions f_8 to f_{13}, the three state-of-the-art DE variants (DE/rand/1/either-or, SADE, and NSDE) and DEGL/SAW outperformed the two classical DE algorithms: DE/rand/1/bin and DE/target-to-best/1/bin. The quality of the solutions produced by the SADE, DE/target-to-best/1/bin, and NSDE algorithm is close to that of the DEGL in a few cases (e.g. the 25-dimensional f_{12}, f_{14}, and the 2-dimensional f_{16} and f_{18} functions).

It is interesting to see that out of the 34 benchmark instances, in 25 cases DEGL outperforms its nearest competitor in a statistically significant fashion. In three cases (f_1 with D=100, f_8 with D=25, f_9 with D=100, and f_{12} with D=25) DE/rand/1/either-or achieved best average accuracy beating DEGL, which remained the second best algorithm. Paired t-tests, however, confirm that the difference of their means is not statistically significant for f_1 and f_9 in 100 dimensions.

Table 7. Average and the standard deviation of the best-of-run solution for 50 independent runs tested on f_9 to f_{21}

Func	D	Max FEs	Mean Best Value (Standard Deviation)						
			DE/rand/1/bin	DE/target-to-best/1/bin	DE/rand/1/either-or	SADE [27]	NSDE [30]	DEGL/SAW	Statistical Significance
f_9	25	5×10^5	1.0453e-03 (8.04e-02)	9.5278e-01 (4.72e-01)	1.7109e-23 (2.726e-24)	6.7381e-24 (3.728e-21)	4.8392e-21 (8.872e-20)	**5.8492e-25 (5.333e-27)**	•
	100	5×10^6	2.1121e-02 (4.86e-03)	6.76249e-01 (4.27e-01)	**8.4719e-23 (9.36e-22)**	5.8824e-21 (4.83e-20)	5.5732e-05 (5.93e-04)	1.7728e-22 (3.88e-20)	•
f_{10}	25	5×10^5	4.1902e-08 (3.36e-08)	9.8035e-03 (6.80e-03)	6.9437e-15 (4.86e-15)	7.8343e-15 (2.85e-15)	5.9749e-10 (3.2231e-04)	**5.9825e-23 (1.00e-22)**	+
	100	5×10^6	7.6687e-05 (6.767e-05)	6.76249e-01 (4.237e-01)	6.9398e-13 (4.852e-13)	3.0665e-12 (5.125e-13)	4.1232e-05 (7.496e-06)	**8.52742e-17 (1.365e-15)**	+
f_{11}	25	5×10^5	6.8318e-22 (3.837e-25)	7.94504e-07 (8.03e-08)	3.0905e-34 (7.462e-34)	1.8274e-28 (7.682e-29)	7.9318e-26 (3.774e-28)	**2.9931e-36 (4.736e-35)**	+
	100	5×10^6	2.1962e-10 (8.45e-11)	5.27284e-05 (4.63e-07)	3.2928e-12 (2.77e-13)	8.9569e-13 (1.02e-14)	5.0392e-10 (4.29e-08)	**4.11464e-15 (6.02e-16)**	+
f_{12}	25	5×10^5	7.0931e-16 (6.22e-15)	2.8962e-13 (2.25e-10)	**5.1469e-32 (4.22e-29)**	9.3718e-24 (6.193e-28)	5.8471e-21 (3.728e-21)	7.2094e-27 (4.838e-28)	+
	100	5×10^6	4.2455e-10 (2.96e-09)	5.21919e-02 (2.94e-04)	2.9137e-15 (4.30e-16)	2.8417e-15 (1.45e-14)	4.8923e-12 (8.45e-13)	**3.00496e-18 (4.82e-17)**	+
f_{13}	25	5×10^5	-1.12836e+00 (4.46e-08)	-4.86485e-01 (1.08e-10)	-1.1382e+00 (3.29e-10)	-1.14280e+00 (3.85e-07)	-1.14276e+00 (3.44e-09)	**-1.14282e+00 (5.81e-06)**	+
	100	5×10^6	2.0621e-02 (5.58e-03)	5.81493e-01 (1.08e-02)	2.19321e+00 (3.32e-01)	-1.1014e+00 (6.98e-03)	-1.10266e+00 (7.84e-05)	**-1.14282e+00 (9.02e-05)**	+
f_{14}	2	5×10^5	9.9813292e-01 (5.42e-10)	9.9860553e-01 (4.26e-03)	**9.9800390e-01 (1.13e-16)**	9.9800884e-01 (1.93e-18)	9.9860346e-01 (1.07e-02)	**9.9800390e-01 (1.15e-18)**	NA
f_{15}	4	5×10^5	4.0361420e-04 (2.81e-04)	4.8242655e-04 (6.41e-05)	3.6734442e-04 (5.13e-05)	3.7044472e-04 (9.82e-07)	3.7320963e-04 (4.33e-03)	**3.7041849e-04 (2.11e-09)**	+
f_{16}	2	5×10^5	-1.029922e+00 (1.82e-08)	-1.031149e+00 (2.44e-08)	-1.031242e+00 (4.98e-06)	**-1.031630e+00 (9.73e-12)**	**-1.031630e+00 (3.33e-10)**	**-1.031630e+00 (4.28e-10)**	NA
f_{17}	2	5×10^5	3.9788959e-01 (6.39e-06)	3.9789793e-01 (6.28e-07)	3.9788915e-01 (6.82e-06)	3.9788783e-01 (2.68e-06)	3.9788392e-01 (4.09e-06)	**3.9788170e-01 (8. 54e-04)**	•
f_{18}	2	5×10^5	3.0834435e+00 (4.73e-01)	3.146090e+00 (5.83e-01)	**3.000000e+00**	**3.000000e+00**	**3.000000e+00**	**3.000000e+00**	NA
f_{19}	2	5×10^5	-1.0042985e+01 (4.32e-05)	-6.840054e+00 (3.87e+00)	-1.010974e+01 (2.67e-05)	-1.015050e+01 (4.59e-04)	-1.014876e+01 (3.57e-03)	**-1.015323e+01 (7.34e-08)**	+
f_{20}	2	5×10^5	-1.0400382e+01 (8.54e-10)	-1.040073e+01 (4.53e-08)	-1.040068e+01 (9.24e-10)	-1.040189e+01 (6.94e-05)	-1.040089e+01 (3.00e-08)	**-1.040295e+01 (5.93e-04)**	+
f_{21}	2	5×10^5	-1.0536082e+01 (2.87e-03)	-7.023436e+01 (4.78e-05)	-1.0474381e+01 (6.88e-03)	-1.0536234e+01 (2.46e-06)	-1.023436e+01 (2.72e-02)	**-1.053641e+01 (3.90e-08)**	+

Table 8. Average and the standard deviation of the best-of-run solution for 50 independent runs tested on composite functions CF1 to CF3 taken from the CEC'05 benchmarks

Func	D	Max FEs	Mean Best Value (Standard Deviation)							
			DE/rand/1 /bin	DE/target-to-best/1/ bin	DE/rand/1/ either-or	SADE [27]	NSDE [30]	DEGL/ SAW (Cr = 0.9)	DEGL/ SAW (Cr = 1)	Statistical Significance
CF1	10	5×10^6	6.400300e+02 (2.3428e+02)	7.92834e+02 (3.0922e+02)	6.280932e+02 (2.0703e+02)	5.334983e+02 (3.9672e+01)	6.230469e+02 (4.5297e+01)	6.19227e+02 (6.8341e+01)	**5.03826e+02 (4.0995e+01)**	+
CF2	10	5×10^6	6.340356e+02 (2.6635e+02)	7.993241e+02 (4.6723e+02)	6.157323e+02 (9.8836e+01)	5.15284e+02 (2.0784e+02)	7.198302e+02 (4.8735e+02)	7.60543e+02 (9.7837e+01)	**4.18542e+02 (8.9984e+01)**	+
CF3	10	5×10^6	8.56392e+02 (9.4863e+01)	1.12873e+03 (6.7394e+01)	7.48427e+02 (5.8473e+01)	7.88492e+02 (4.4342e+01)	8.93824e+02 (3.8764e+01)	6.74823e+02 (5.8471e+01)	**4.76239e+02 (3.7842e+01)**	+

As long as $Cr < 1$, DEGL will not be rotationally invariant, i.e., its performance will depend on the orientation of the coordinate system in which vectors are evaluated [3]. Since the composite functions CF1, CF2 and CF3 are rotated in nature, we also solve them using DEGL/SAW with $Cr = 1$. Table 5 shows that this rotationally invariant version of DEGL performs significantly better on the composite test functions as compared to the DEGL with $Cr = 0.9$. However, the performance over the 21 traditional benchmarks (which are unrotated) is nearly the same for both the versions. In order to save space we have not shown the results of DEGL/SAW with $Cr = 1$ in Tables 6 and 7.

6.2.2 Comparison of the Convergence Speed and Success Rate

In order to compare the speeds of different algorithms, we select a threshold value of the objective function for each benchmark problem. For functions with minima at 0, this threshold is at 10^{-20}. To obtain an unbiased comparative performance, for other functions, this value is chosen to be somewhat larger than the minimum objective function value found by each algorithm in Tables 6, 7, and 8. We run each algorithm on a function and stop as soon as the best fitness value determined by the algorithm falls below the predefined threshold. Then we note the number of FEs the algorithm takes. A lower number of FEs corresponds to a faster algorithm. Tables 9, 10, and 11 report the number of runs (out of 50) that managed to find the optimum solution (within the given tolerance) as well as the mean number of FEs and standard deviations (within parenthesis) required by the algorithms to converge within the prescribed threshold value. Entries marked as 0 indicate that no runs of the corresponding algorithm converged below the threshold objective function value. Missing values of standard deviation in these tables also indicate a zero standard deviation.

Tables 6 and 9 indicate that, not only does DEGL/SAW yield the most accurate results for nearly all the benchmark problems, but it does so consuming the least amount of computational time. In addition, the number of runs that converge below a pre-specified cut-off value is also greatest for DEGL over most of the benchmark problems covered here. This indicates the higher robustness (i.e. the ability to produce similar results over repeated runs on a single problem) of the algorithm as compared to its other four competitors. Usually in the community of stochastic search algorithms, robust search is weighted over the highest possible convergence rate [56, 57].

Table 9. No. of successful runs, mean no. of FEs and standard deviation (in parentheses) required to converge to the cut-off fitness over the successful runs for functions f_8 to f_{11}.

Func tion	D	Threshold objective function value	No. of successful runs, mean no. of FEs and (standard deviation) required to converge to the prescribed threshold fitness					
			DE/rand/1 /bin	DE/target-to-best/1/ bin	DE/rand/1 /either-or	SADE [28]	NSDE [31]	DEGL/ SAW
f_1	25	1.00e-20	50, 109372.5 (4773.28)	50, 376421.20 (10983.46)	50, 98204.24 (2942.87)	50, 104982.64 (5182.67)	50, 105727.80 (3427.57)	**50, 91935.40 (3888.45)**
	100	1.00e-20	50, 687322.24 (12153.67)	50, 1033567.40 (58391.56)	**50, 403922.56 (3814.25)**	50, 738720.84 (28731.88)	50, 565382.24 (2827.56)	50, 498521.54 (10832.41)
f_2	25	1.00e-20	50, 266371.40 (31923.45)	50, 417382.80 (23221.45)	50, 198342.22 (3421.68)	50, 306742.28 (18534.55)	50, 300371.48 (9034.26)	**50, 157234.76 (4451.72)**
	100	1.00e-20	13, 2034583.46 (18235.48)	6, 2935411.45 (21893.56)	28, 1062744.69 (44583.41)	23, 1257362.57 (3417.34)	20, 1782336.10 (36710.05)	**34, 978357.83 (23727.45)**
f_3	25	1.00e-20	12, 298341.67 (24376.27)	5, 378392.20 (34621.22)	50, 123682.54 (63827.06)	16, 296473.93 (27268.45)	7, 363986.82 (52741.78)	**50, 110528.68 (13873.51)**
	100	1.00e-20	13, 2638224.33 (57398.21)	10, 4562312.70 (17372.68)	15, 2745218.47 (37123.69)	14, 2696359.51 (14225.47)	13, 2671982.93 (46188.26)	**18, 2063728.48 (27351.57)**
f_4	25	1.00e-20	16, 376291.47 (12836.48)	8, 467262.25 (26111.78)	19, 309309.52 (17829.46)	17, 292478.83 (8372.58)	11, 408291.79 (26721.77)	**21, 294812.82 (36173.52)**
	100	1.00e-20	19, 3174782.17 (17283.49)	3, 4453782.67 (18253.58)	22, 3228379.27 (4824.81)	17, 3139382.38 (33728.42)	5, 4140835.40 (22338.86)	**25, 2263976.44 (28371.46)**
f_5	25	1.00e-20	50, 356253.38 (82732.33)	17, 478290.91 (57263.72)	50, 315633.92 (47192.57)	**50, 267319.74 (23556.24)**	50, 299831.26 (48382.57)	50, 338279.08 (28846.37)
	100	1.00e-20	1, 3398272	0	50, 3067263.78 (56723.83)	50, 2844738.62 (66729.38)	3, 4563742.33 (128123.57)	**50, 2709313.82 (12338.11)**
f_6	25	1.00e-20	50, 189367.38 (83412.84)	50, 132676.28 (6769.48)	50, 122845.64 (7378.36)	50, 173490.18 (7638.46)	50, 235177.72 (13223.94)	**50, 96832.24 (4631.66)**
	100	1.00e-20	18, 2357827.59 (33253.68)	16, 3098277.26 (83921.47)	20, 2299868.50 (27632.58)	47, 1824359.69 (27733.61)	25, 3622719.24 (47378.19)	**50, 1238461.98 (36278.64)**
f_7	25	1.00e-20	0	0	2, 467236.50 (43827.83)	0	0	**4, 417823.25 (27192.82)**
	100	1.00e-20	0	0	1, 3689267.48	0	0	**3, 3163563.67 (78282.58)**
f_8	25	-1.0410e +04	12, 19817.50 (8723.837)	17, 13039.65 (336.378)	50, 12410.04 (1201.278)	50, 9887.50 (822.281)	32, 37847.82 (4431.90)	**50, 9492.64 (871. 76)**
	100	-4.1800e +04	3, 359834.33 (4353.825)	1, 51729	13, 133282.73 (5362.366)	25, 363291.80 (2338.944)	20, 2178283.50 (24332.78)	**35, 39928.45 (231.627)**
f_9	25	1.00e-20	19, 345328.18 (41128.91)	13, 46843.92 (34521.372)	50, 330272.74 (3642.289)	50, 195823.88 (4249.392)	44, 345654.73 (326.84)	**50, 87148.34 (1325.72)**
	100	1.00e-20	5, 1840322.80 (3852.196)	2, 2022275.50 (27327.24)	50, 838932.48 (23677.66)	50, 744938.28 (34147.928)	16, 3290384.57 (53209.58)	**50, 539282.72 (26547.09)**
f_{10}	25	1.00e-20	14, 226816.89 (44721.76)	4, 412675.25 (16834.37)	34, 238372.74 (32325.67)	32, 236290.86 (15533.08)	26, 287812.83 (14039.54)	**50, 224883.78 (13212.87)**
	100	1.00e-20	13, 1873625.56 (29123.902)	2, 4486372.50 (98273.57)	15, 1782210.66 (72233.371)	13, 1065920.64 (24383.71)	7, 2082983.84 (81744.84)	**27, 925628.73 (7823.28)**
f_{11}	25	1.00e-20	50, 333948.52 (12314.821)	6, 356061.52 (11300.97)	50, 225092.84 (12123.19)	50, 316382.04 (35338.83)	50, 369283.71 (45478.88)	**50, 196258.22 (14235.83)**
	100	1.00e-20	26, 1887635.65 (44612.34)	12, 2833416.96 (17218.06)	29, 2633782.74 (10217.26)	34, 1936287.62 (14235.37)	27, 2235653.56 (30362.67)	**43, 1627092.58 (11217.31)**

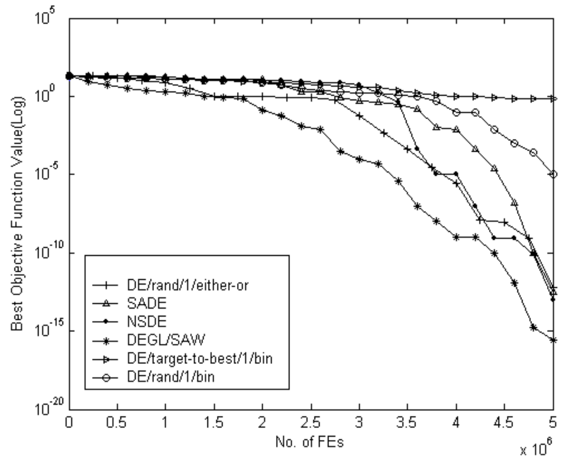
Table 10. No. of successful runs, mean no. of FEs and standard deviation (in parentheses) required to converge to the cut-off fitness over the successful runs for functions f_{12} to f_{21}.

Func	D	Threshold objective function value	No. of successful runs, mean no. of FEs and (standard deviation) required to converge to the prescribed threshold fitness					
			DE/rand/1 /bin	DE/target-to-best/1/ bin	DE/rand/1/ either-or	SADE [28]	NSDE [31]	DEGL/ SAW
f_{12}	25	1.00e-20	35, 294584.44 (22563.378)	30, 3472185.67 (13382.229)	42, 209372.87 (12742.03)	50, 126574.64 (16833.89)	46, 478732.05, (3884.04)	**50, 150039.62 (4831.28)**
	100	1.00e-20	8, 3122658.25 (62922.84)	5, 3908138.80 (13937.383)	23, 2664722.53 (47212.38)	20, 1637409.40 (18219.526)	10, 2673864.70 (53121.65)	**27, 1436190.89 (13627. 82)**
f_{13}	25	-1.1428e+00	13, 230372.52 (7313.297)	3, 428023.33 (84517.371)	26, 237639.09 (14573.96)	42, 213739.78 (12347.391)	24, 738742. 34 (24322.82)	**48, 121940.72 (33398.90)**
	100	-1.1428e+00	1, 3328426	0	0	14, 1702654.85 (21743.57)	25, 1283665.44 (9487.37)	**28, 398493.74 (25134.38)**
f_{14}	2	9.9800390e -01	19, 94233.57 (2312.57)	14, 89371.53 (1409.26)	46, 68392.37 (5231.48)	27, 84032.58 (3842.53)	15, 77362.94 (4437.28)	**47, 67823.84 (3725.36)**
f_{15}	4	3.705e-04	0	0	13, 58935.28 (3822.72)	33, 68293.46 (2219.58)	20, 73821.05 (6319.48)	**41, 65783.38 (1749.51)**
f_{16}	2	-1.03170e+00	32, 83920.68 (2124.56)	37, 98529.61 (1098.59)	27, 83782.79 (1271.47)	50, 77129.34 (3731.63)	50, 71036.28 (1211.48)	**50, 67382.39 (1726.49)**
f_{17}	2	3.980e-01	41, 103273.57 (2231.68)	43, 79382.42 (907.31)	43, 75823.45 (3281.68)	38, 78939.37 (1325.46)	47, 84983.94 (2258.10)	**49, 73727.83 (4308.58)**
f_{18}	2	3.00e+00	21, 67392.59 (3381.62)	23, 77539.42 (4839.86)	**50, 89482.78 (3238.56)**	**50, 79035.28 (3381.98)**	**50, 80382.70 (419.49)**	**50,** 69837.62 **(1724.08)**
f_{19}	2	-1.01550e+01	23, 109372.48 (3341.67)	34, 98922.93 (3212.68)	44, 68672.70 (1332.67)	41, 67478.37 (2001.83)	37, 79820.42 (1692.78)	**46, 58372.96 (3827.58)**
f_{20}	2	-1.04500e+01	35, 84721.07 (3412.39)	42, 107482.69 (10824.57)	48, 58373.47 (2221.680	47, 48372.83 (2294.83)	44, 85933.58 (3329.74)	**50, 56098.08 (3187.44)**
f_{21}	2	-1.05500e+01	26, 86743.93 (6983.07)	30, 85999.67 (2901.83)	32, 84892.66 (2319.59)	46, 68492.69 (2326.09)	23, 100232.67 (3721.78)	**49, 67583.93 (3317.58)**

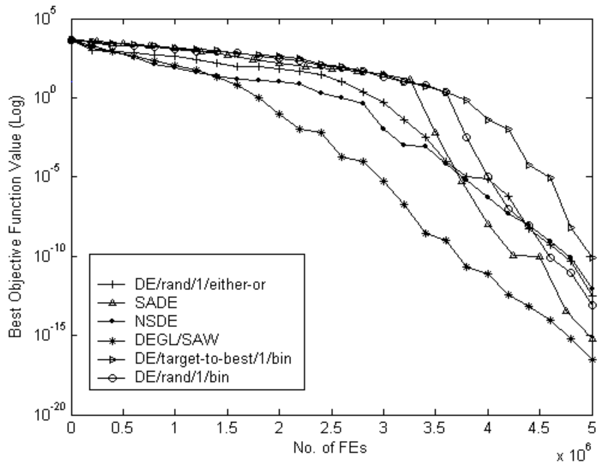
Table 11. No. of successful runs, mean no. of FEs and standard deviation (in parentheses) required to converge to the cut-off fitness over the successful runs for composite functions CF1 to CF3.

Func	D	Threshold objective function value	No. of successful runs, mean no. of FEs and (standard deviation) required to converge to the prescribed threshold fitness						
			DE/rand/1 /bin	DE/target-to-best/1/ bin	DE/rand/1/ either-or	SADE [28]	NSDE [31]	DEGL/ SAW (Cr = 0.9)	DEGL/ SAW (Cr = 1)
CF1	10	8.10e+02	34, 2683073.04 (45214.48)	19, 3835238.75 (18183.95)	42, 637222.35 (39357. 23)	50, 1823847.64 (52932.821)	12, 624732.56 (35330.493)	36, 1707873.04 (13434.482)	**50, 1645938.75 (18843.905)**
CF2	10	8.10e+02	33, 530857.85 (13439.09)	21, 2539841.89 (87438.490)	25, 942325.40 (3173.74)	25, 818472.16 (7384.492)	37, 510932.79 (3438.473)	36, 1230857.85 (13139.409)	**39, 83401.86 (5438.46)**
CF3	10	1.20e+03	17, 3645817.50 (95823.83)	17, 4834039.65 (35336.78)	41, 1597232.03 (37811.28)	40, 196887.50 (12372.28)	24, 3139492.64 (54431.26)	45, 149817.56 (2339.37)	**50, 913039.68 (3576.78)**

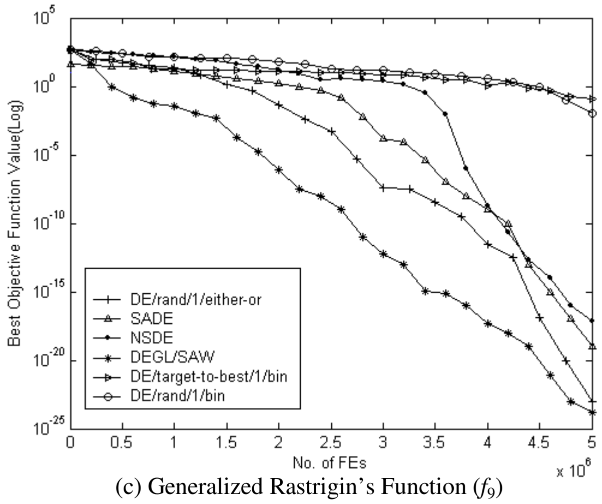
The convergence characteristics of seven difficult test functions are shown in Figure 7 in terms of the fitness value of the median run of each algorithm. All the graphs except for the composite functions CF1 to CF3 have been drawn for D = 100 dimensions. Convergence graphs for the composite functions appear for D = 10 dimensions.



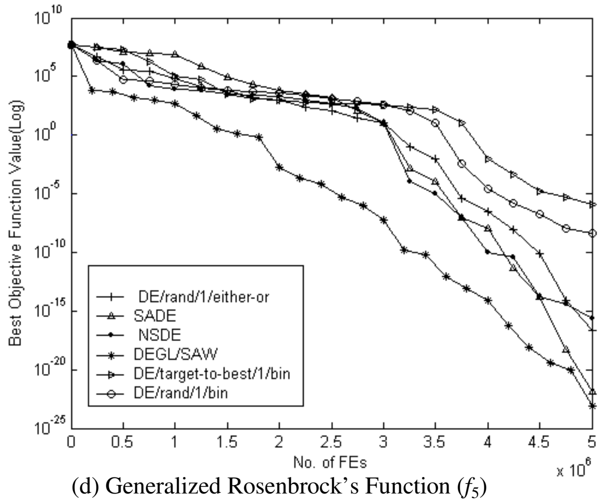
(a) Generalized Ackley's Function (f_{10})



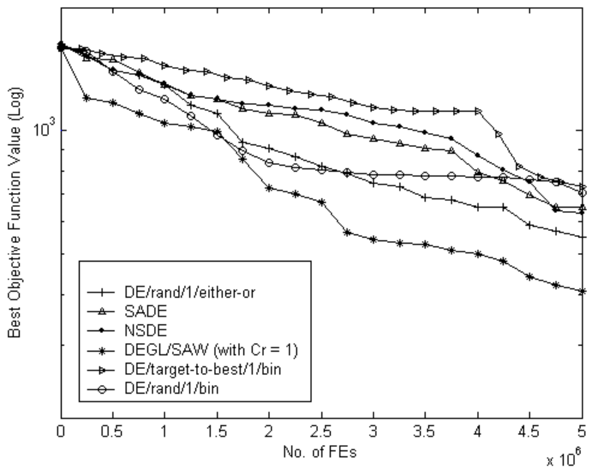
(b) Generalized Griewnk's Function (f_{11})



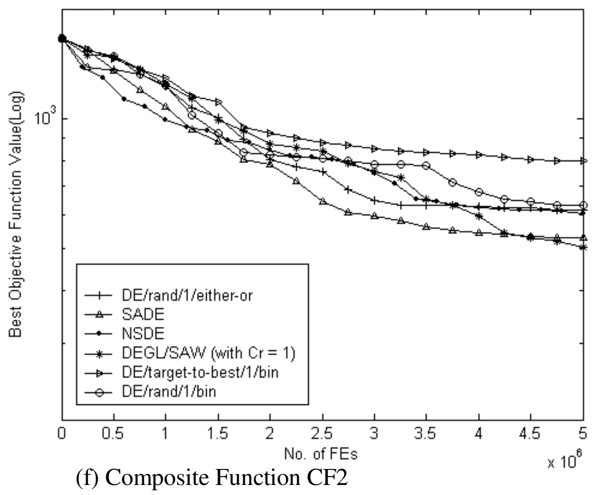
(c) Generalized Rastrigin's Function (f_9)



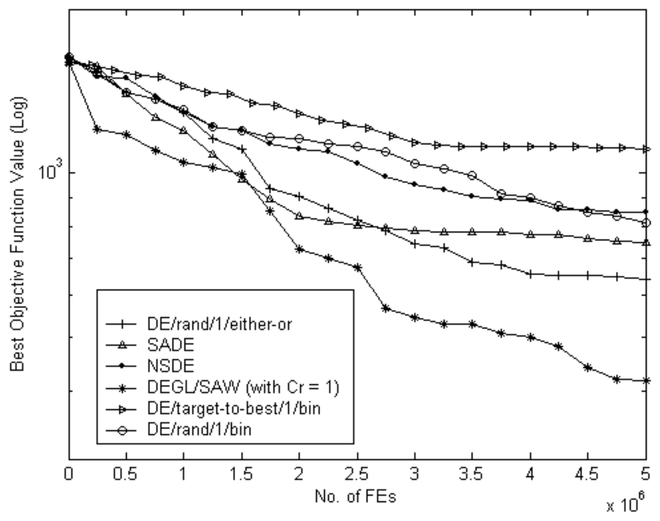
(d) Generalized Rosenbrock's Function (f_5)



(e) Composite Function CF1



(f) Composite Function CF2



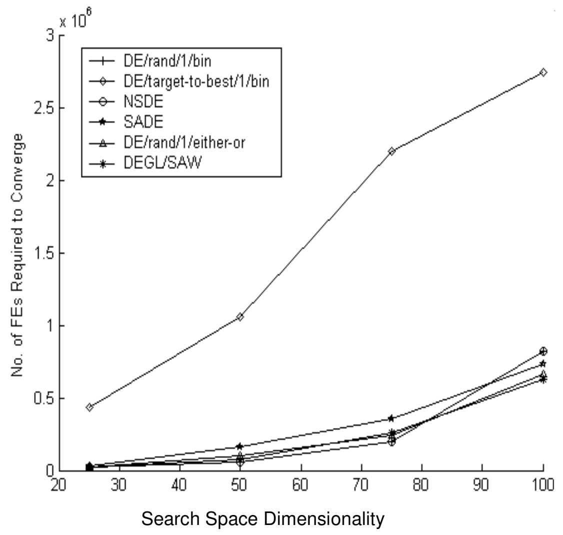
(g) Composite Function CF3

Fig. 7. Progress towards the optimum solution for median run of six algorithms over seven difficult test functions.

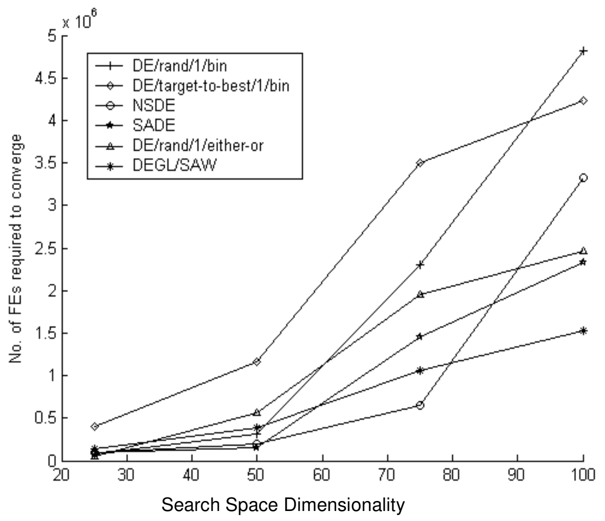
6.2.3 Scalability Comparison

Performance of most of the evolutionary algorithms (including DE and PSO) deteriorates with the growth of the dimensionality of the search space. Increase of dimensions implies a rapid growth of the hyper-volume of the search space and this in turn slows down the convergence speed of most of the global optimizers. Here we show how the performance of the six DE variants scale against the growth of dimensions from 25 to 100. Figure 8 shows the scalability of the six algorithms over four difficult test functions - how the average computational cost (measured in number of FEs required to yield a threshold fitness value) to find the solution varies with an increase in the dimensionality of the search space.

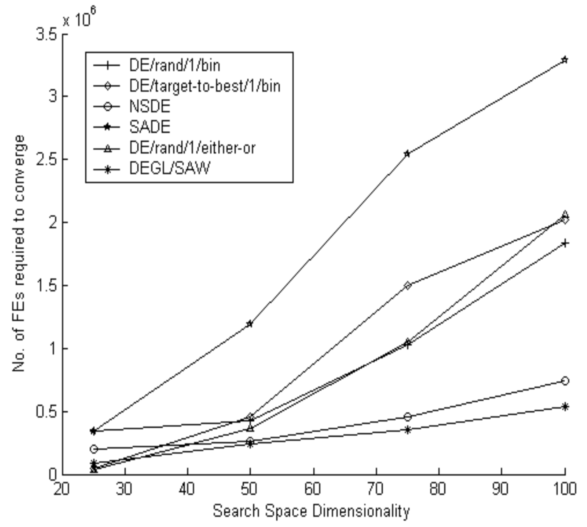
We note that the computational cost of both DEGL/SAW and SADE (to yield a given accuracy) increases most sluggishly with the search space dimensionality for the following test-functions: f_5, f_{10}, f_{11}, and f_9.



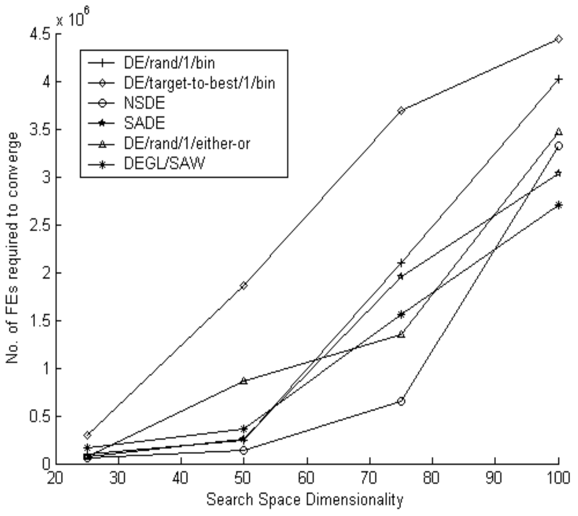
(a) Generalized Ackley's Function (f_{10})



(b) Generalized Griewank's Function (f_{11})



(c) Generalized Rastrigin's Function (f_9)



(d) Generalized Rosenbrock's Function (f_5)

Fig. 8. Variation of mean number of FEs required for convergence to predefined threshold accuracy with increase in dimensionality of the search space.

6.3 Comparison with other State-of-the-art Evolutionary Techniques

In this section we compare the performance of DEGL/SAW with that of four state-of-the-art evolutionary and swarm-based optimization techniques, well-known as CPSO-H [38], IPOP-CMA-ES [58], MA-S2 [59], and G3 with PCX [60]. Below we briefly describe each of these algorithms.

1) CPSO-H: van den Bergh and Engelbrecht proposed a Cooperative Particle Swarm Optimizer (CPSO) in [36]. Although CPSO uses one-dimensional (1-D) swarms to search each dimension separately, the results of these searches are integrated by a global swarm to significantly improve the performance of the original PSO on multi-modal problems. The CPSO-H algorithm uses a hybrid swarm, consisting of a maximally split cooperative swarm (*D* one-dimensional swarms for one *D*-dimensional parameter vector) and a plain swarm. Both components employ identical values for the acceleration coefficients ($C_1 = C_2 = 1.49$) and the inertial factor ω decreasing linearly with time. They use a maximum velocity $\vec{V}_{\max}$ clamped to the search domain [38].

2) IPOP-CMA-ES: CMA-ES (Covariance Matrix Adaptation Evolution Strategy) [61, 62] is an evolutionary strategy that uses informed mutation based on local structural information, but does not directly bias its search motion toward other individuals of the population. Auger and Hansen have recently proposed a restart CMA-ES [58], where the population size is increased (IPOP) for each restart. By increasing the population size, the search characteristic becomes more global after each restart. This variant is named as IPOP-CMA-ES.

3) MA-S2: Memetic Algorithms (MAs) [63, 64] are based on the hybridization of Genetic Algorithm (GA) with Local Search (LS) techniques. In this study, MA-S2 [59] stands for an adaptive Meta-Lamarckian learning-based MA that employs a stochastic approach (the biased roulette wheel strategy) making use of the knowledge gained online to select a suitable local method with the GA.

4) G3 with PCX: The main research effort in the field of real parameter GA is more or less focussed on the design of efficient recombination operators used to create offspring from parent solutions. Deb *et al.* [60] proposed a generic parent-centric recombination scheme (PCX) and integrated it with a steady state, elite preserving, scalable, and computationally fast population alteration model of the GA, which they named the G3 (Generalized Generation Gap) model. Their results indicate that the G3 model with PCX can outperform many other existing GA models when tested on the standard benchmark functions.

We employ the best parametric set-up for all these algorithms as prescribed in their respective sources. The mean and the standard deviation (within parentheses) of the best-of-run values of 50 independent runs for each algorithm have been presented in Tables 12 and 13. In order to save space we report only the hardest problem instances (multi-dimensional functions with D = 100) in these tables. The algorithms compared in this section have different population sizes and also differ in their initial population structure. Thus to test the statistical significance of the results, we used two-tailed unpaired t tests between the two best algorithms. The results of t test have been indicated in the 9-th column of Table 12 and 10-th column of Table 13. Note that here '+' indicates the t value of 98 degrees of freedom is significant within a 95% confidence interval by two-tailed test, '.' means the difference of means is not statistically significant and 'NA' stands for Not Applicable, covering cases in which two or more algorithms achieve the best accuracy results.

These simulation results show that DEGL/SAW is superior to all the other algorithms in terms of the average final accuracy over 12 cases reported in Table 12 and 2 cases in Table 13. DEGL/SAW yields results comparable to two or more algorithms for 6 cases in Table 12. It is interesting to see that out of the 12 cases in Table 12, where DEGL/SAW was able to beat all its contestant algorithms, for 9 instances the difference between the means of DEGL/SAW and its nearest competitor is statistically significant. From Table 12, we find that CPSO-H was able to outperform DEGL/SAW (and all the other contestants) over the 100-dimensional Schwefel's problem 1.2 (f_3) and IPOP-CMA-ES alone achieved the greatest accuracy for the 100-dimensional generalized penalized function (f_{12}) beating DEGL/SAW. For f_3, DEGL/SAW remained the third best algorithm (after CPSO-H and G3 with PCX) while for the f_{12} function, it secured the second place in terms of final accuracy. However, the last column of Table 12 shows the difference of means of DEGL/SAW and IPOP-CMA-ES is *not* statistically significant in the case of the f_{12} function.

For lower dimensional multi-modal functions f_{14} to f_{21}, almost all the algorithms end up with nearly equal levels of final accuracy, f_{19} being an exception where DEGL/SAW appeared to perform significantly better as compared to all other algorithms. For the higher dimensional and multi-modal functions f_8 to f_{13}, however, CPSO-H and IPOP-CMA-ES remained as the toughest competitor of DEGL/SAW. Note that over these functions DEGL/SAW remained statistically better as compared to the MA-S2 algorithm, which also employs local search strategies in an adaptive fashion with GA. Except for the generalized penalized function f_{12}, DEGL/SAW met or beat the IPOP-CMA-ES over all other multi-modal functions in 100 dimensions. The final accuracy provided by DEGL/SAW improves significantly as compared to all other algorithms for three hardest uni-modal functions: the generalized Rosenbrock's function (f_5), the discontinuous step function (f_6), and the noisy quartic function (f_7).

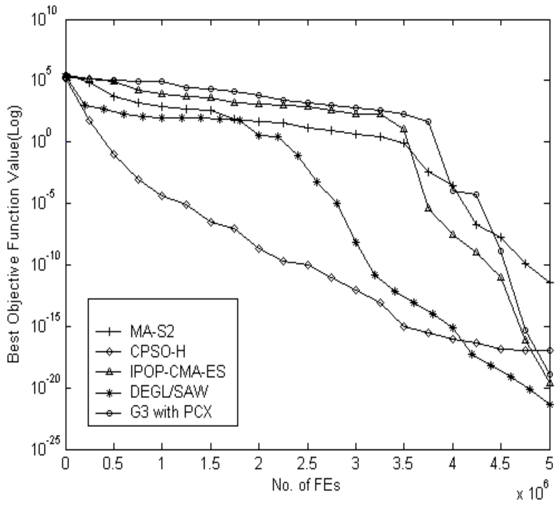
The convergence characteristics of the contestant algorithms over the six hardest test functions have been shown in Figure 9 in terms of the objective function value of the median run of each algorithm. For the step function, characterized by plateaus and discontinuity, DEGL/SAW maintained a steady convergence rate that finally finished at the lowest objective function value, while the local search-based MA-S2 showed a much slower convergence. Usually a local search method that relies on geographical neighborhoods performs poorly on the step function because the algorithm mainly searches in a relatively small local neighborhood. On the other hand, DEGL employs a geographically randomized neighborhood structure (*local* only in the sense of vector indices), and the individuals can make longer jumps enabling them to move from one plateau to a lower one with relative ease.

Table 12. Average and standard deviation of the best-of-run solutions for 50 independent runs and the success rate tested on f_1 to f_{21}

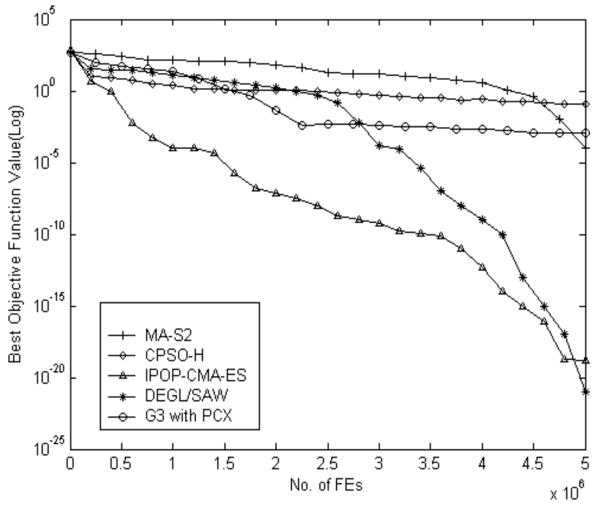
Func	D	Max FEs	Mean Best Value (Standard Deviation)					
			CPSO-H	IPOP-CMA-ES	MA-S2	G3 with PCX	DEGL/SAW	Statistical Significance
f_1	100	5×10^6	6.5635e-22 (7.234e-28)	9.6853e-23 (7.232e-26)	7.5364e-22 (3.454e-25)	2.8002e-20 (6.467e-14)	**8.3812e-23 (3.925e-25)**	.
f_2	100	5×10^6	7.4164e-08 (6.225e-07)	2.7429e-03 (1.648e-07)	6.2899e-04 (1.91e-15)	2.6595e-06 (3.36e-10)	**9.1395e-10 (3.36e-10)**	+
f_3	100	5×10^6	**3.5712e-23 (7.239e-22)**	2.5358e-08 (1.923e-09)	8.0005e-07 (8.947e-05)	3.7659e-10 (2.596e-10)	9.7852e-10 (6.132e-08)	+
f_4	100	5×10^6	6.5132e-13 (1.795e-16)	1. 7685e-12 (4.949e-06)	4.8865e-12 (2.209e-13)	7.4823e-13 (3.773e-09)	**3.7068e-14 (1.08e-12)**	.
f_5	100	5×10^6	1.5041e-01 (9.423e-01)	6.0499e-22 (8.345e-24)	1.5639e-20 (2.700e-20)	5.7778e-18 (2.233e-19)	**1.5463e-25 (7.301e-22)**	+
f_6	100	5×10^6	1.4532e-15 (1.713e-16)	2.1052e-20 (8.691e-21)	1.4455e-13 (3.938e-11)	7.0054e-17 (2.644e-14)	**8.6493e-22 (8.483e-23)**	+
f_7	100	5×10^6	8.5829e-13 (1.492e-03)	2.9890e-03 (7.086e-01)	9.6648e-05 (2.331e-09)	1.7984e-02 (6.834e-03)	**6.9921e-06 (4.56e-05)**	+
f_8	100	5×10^6	-4.0572e+04 (9.481e-06)	-4.18783e+04 (1.129e-04)	-4.18774e+04 (4.227e-05)	-4.03386e+04 (2.349e-05)	**-4.18983e+04 (6.98e-06)**	+
f_9	100	5×10^6	1.7382e-01 (4.093e-02)	9.24702e-21 (4.324e-21)	7.32562e-04 (2.781e-05)	5.92381e-03 (3.779e-04)	**1.7728e-22 (3.838e-23)**	+
f_{10}	100	5×10^6	1.7725e-12 (2.489e-13)	8.85280e-17 (7.638e-14)	3.71596e-09 (9.328e-08)	3.47432e-10 (7.146e-09)	**3.52742e-17 (1.365e-15)**	+
f_{11}	100	5×10^6	2.5361e-02 (7.2281e-03)	3.67528e-14 (6.932e-14)	1.56794e-13 (3.6433e-09)	8.92369e-11 (8.157e-15)	**4.11464e-15 (6.02e-16)**	+
f_{12}	100	5×10^6	4.2042e-10 (6.955e-11)	**4.45366e-19 (3.634e-16)**	2.75934e-09 (8.359e-06)	6.86492e-04 (8.035e-03)	8.00496e-19 (4.82e-17)	.
f_{13}	100	5×10^6	**-1.142822e+00 (9.472e-06)**	**-1.142822e+00 (1.342e-03)**	-1.00864e+00 (1.44e-05)	-1.10967e+00 (8.345e-01)	**-1.142823e+00 (9.032e-05)**	NA
f_{14}	2	5×10^6	**9.9800390e-01 (7.228e-16)**	**9.9800390e-01 (2.673e-16)**	9.9800400e-01 (9.373e-09)	**9.9800390e-01 (1.138e-16)**	**9.9800390e-01 (1.15e-18)**	NA
f_{15}	4	5×10^6	3.706461e-04 (1.551e-06)	**3.7041849e-04 (4.837e-10)**	3.706851e-04 (2.558e-05)	4.156548e-04 (2.981e-04)	**3.7041849e-04 (2.11e-09)**	NA
f_{16}	2	5×10^6	**-1.031630e+00 (7.236e-11)**	**-1.031630e+00 (3.668e-11)**	-1.031628e+00 (4.538e-08)	**-1.031630e+00 (2.548e-09)**	**-1.031630e+00 (1.749e-10)**	NA
f_{17}	2	5×10^6	3.9788231e-01 (2.683e-06)	**3.9788170e-01 (1.260e-08)**	3.9788794e-01 (7.638e-06)	3.9788396e-01 (6.039e-06)	**3.9788170e-01 (8. 544e-04)**	NA
f_{18}	2	5×10^6	**3.000000e+00**	**3.000000e+00**	**3.000000e+00**	**3.000000e+00**	**3.000000e+00**	NA
f_{19}	2	5×10^6	-1.015306e+00 (2.453e-06)	-1.015314e+01 (8.071e-07)	-1.015058e+01 (1.593e-06)	-1.014888e+01 (5.568e-01)	**-1.015323e+01 (7.341e-08)**	+
f_{20}	2	5×10^6	-1.040236e+01 (3.116e-06)	-1.040293e+01 (7.974e-10)	-1.040125e+01 (1.944e-05)	-1.040089e+01 (3.00e-08)	**-1.040295e+01 (5.923e-04)**	.
f_{21}	2	5×10^6	-1.053427e+01 (1.593e-08)	**-1.053641e+01 (6.049e-07)**	-1.053669e+01 (1.446e-03)	-1.023386e+01 (9.638e-02)	**-1.053641e+01 (3.90e-08)**	NA

Table 13. Average and standard deviation of the best-of-run solutions for 50 independent runs tested on composite functions CF1 to CF3 taken from the CEC'05 benchmarks

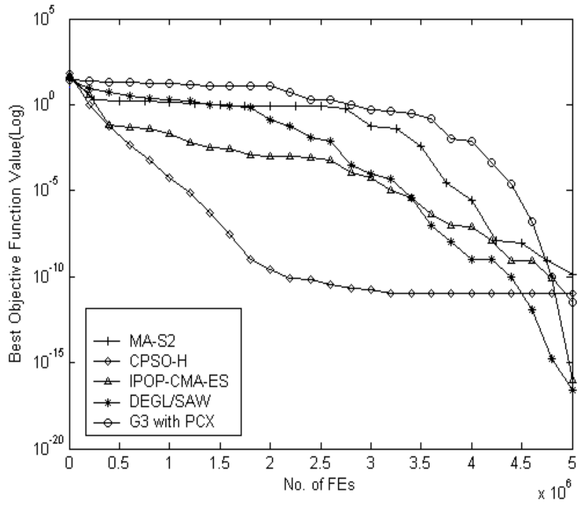
Func	D	Max FEs	Mean Best Value (Standard Deviation)						
			CPSO-H	IPOP-CMA-ES	MA-S2	G3 with PCX	DEGL/SAW ($Cr = 0.9$)	DEGL/SAW ($Cr = 1$)	Statistical Significance
CF1	10	5×10^6	5.24167e+02 (1.046e+01)	**3.83592e+02 (1.236e+02)**	1.98661e+03 (2.123e+02)	1.847894e+03 (3.353e+02)	6.19227e+02 (6.8341e+01)	5.03826e+02 (4.0995e+01)	+
CF2	10	5×10^6	9.23762e+02 (6.718e+01)	6.82114e+02 (1.8469e+01)	1.53459e+03 (1.133e+02)	1.49463e+04 (7.846e+02)	7.60543e+02 (9.7837e+01)	**4.18542e+02 (8.9984e+01)**	+
CF3	10	5×10^6	7.58269e+02 (9.462e+02)	5.12504e+02 (2.586e+02)	7.16728e+02 (2.836e+02)	1.91423e+03 (2.643e+02)	6.74823e+02 (5.8471e+01)	**4.76239e+02 (3.7842e+01)**	+



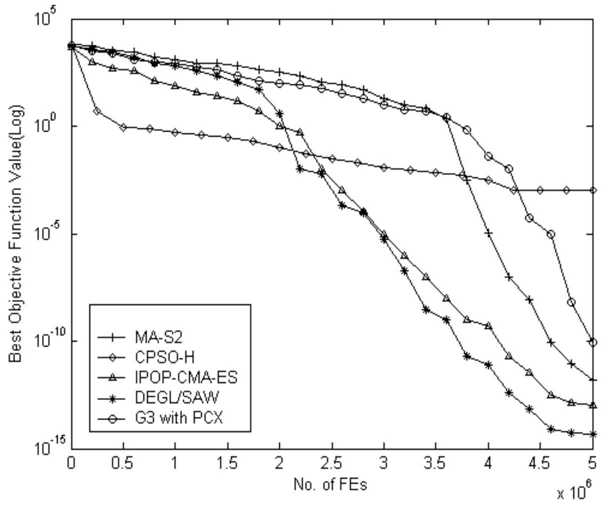
(a) Step Function (f_6)



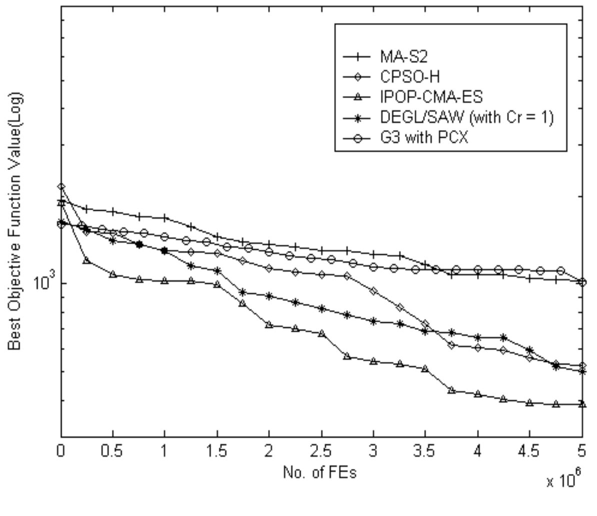
(b) Generalized Rastrigin's Function (f_9)



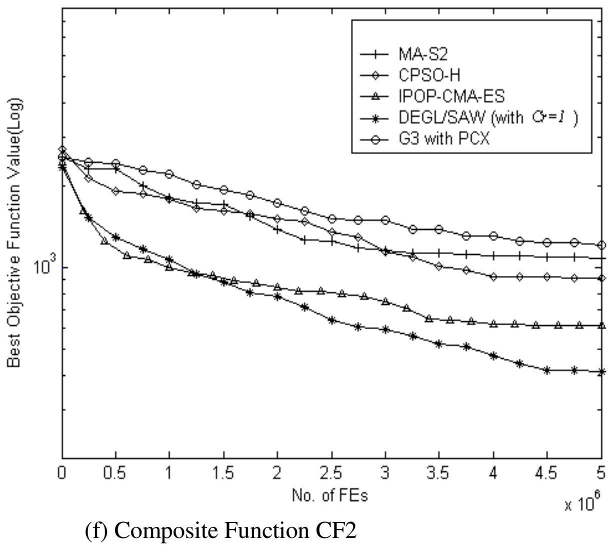
(c) Generalized Ackley's Function (f_{10})



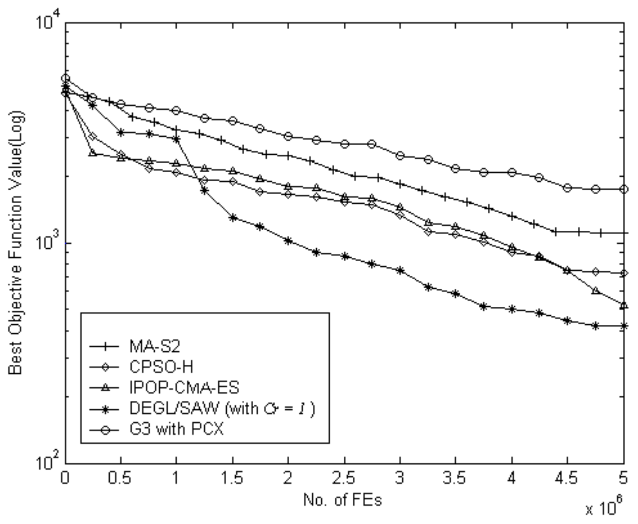
(d) Generalized Griewnk's Function (f_{11})



(e) Composite Function CF1



(f) Composite Function CF2



(g) Composite Function CF3

Fig. 9. Convergence characteristics for median run of five algorithms over seven difficult benchmark functions.

Figure 9 reveals that for Ackley (f_{10}), Rastrigin (f_9), and Griewank (f_{11}), as well as harder composite functions CF1 and CF2, initially CPSO-H and IPOP-CMA-ES converge at the quickest rate among all the algorithms. However, in the neighborhood of the global optima, DEGL/SAW overtakes both of them, attaining greater final accuracy. The composite function CF1 appears as an exception to this trend (that is also exhibited by the convergence graphs of other functions, which were omitted to save space), where the convergence rate of CMA-ES remained higher than DEGL/SAW until the maximum number of FEs were reached.

6.4 Comparative Performance over Real-life Optimization Problems

This section investigates the performance of the six competitive DE-variants over two real-world optimization problems viz. the spread spectrum radar poly-phase code design problem and the sound frequency modulator synthesis problem. Both problems have been briefly described earlier in Section 5.2.

In Table 14, we show the mean and the standard deviation (within parentheses) of the best-of-run values for 30 independent runs of each of the six algorithms over the two most difficult instances of the radar poly-phase code design problem (for dimensions D = 19 and D = 20). Table 15 reports the results of the same experiments performed over the FM synthesizer problem. Figures 9 and 10 graphically present the rate of convergence of the DE-variants for these two problems (graphs in Figure 9 have been shown for 20 dimensions for the radar code design problem). The 8-th column in Table 14 and the 7-th column in Table 15 indicate the statistical significance level obtained from a paired t test between the best and the next-to-best performing algorithms in each case.

Table 14. Average and standard deviation (in parentheses) of the best-of-run solutions for 30 runs over the spread spectrum radar poly-phase code design problem (number of dimensions D = 19 and D = 30). For all cases each algorithm was run up to 5×10^6 FEs.

D	Mean best-of-run solution (Std Dev)						
	DE/rand/1 /bin	DE/target-to-best/1/ bin	DE/rand/1/ either-or	SADE [28]	NSDE [31]	DEGL/ SAW	Statistical Significance
19	7.4849e-01 (8.93e-03)	7.6535e-01 (5.93e-04)	7.5834e-01 (9.56e-04)	7.5932e-01 (3.88e-05)	7.6094e+01 (4.72e-03)	**7.4439e-01 (5.84e-04)**	.
20	8.5746e-01 (4.83e-03)	9.3534e-01 (4.55e-02)	8.3982e-01 (3.98e-03)	8.3453e-01 (6.53e-04)	8.4283e-01 (3.44e-02)	**8.0304e-01 (2.73e-03)**	+

Table 15. Average and standard deviation (in parentheses) of the best-of-run solutions for 50 runs of six algorithms on the frequency modulator synthesis problem. Each algorithm was run for 10^5 FEs.

Mean best-of-run solution (Std Deviation)						
DE/rand/1 /bin	DE/target-to-best/1/ bin	DE/rand/1/ either-or	SADE	NSDE	DEGL/ SAW	Statistical Significance
1.7484e-01 (4.268e-02)	1.8255e+00 (1.158e-01)	3.8523e-04 (2.995e-04)	7.8354e-02 (5.8254e-03)	9.4559e-03 (6.924e-01)	**4.8152e-09 (6.2639e-08)**	+

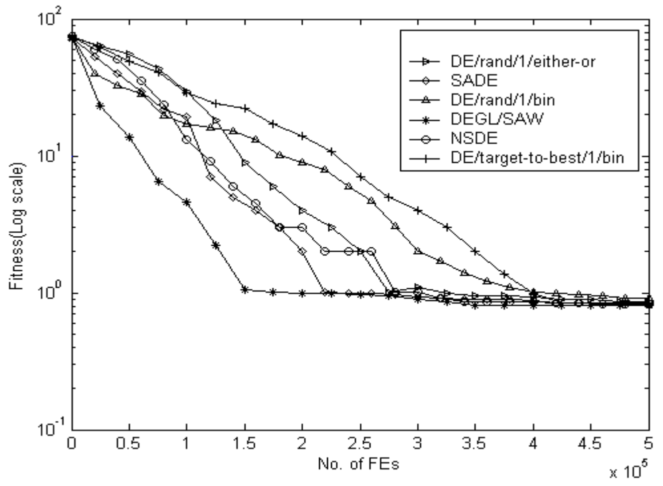


Fig.10. Progress to the optimum solution for spread spectrum radar poly-phase code problem (D = 20).

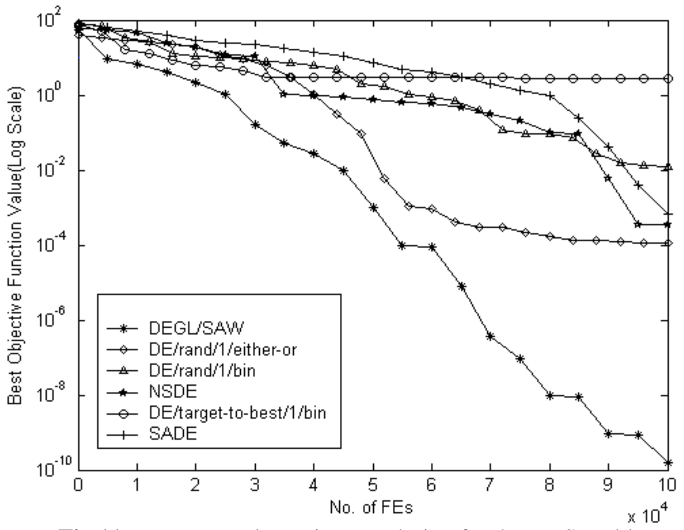


Fig.11. Progress to the optimum solution for the FMS problem.

Tables 14 and 15 show that DEGL/SAW outperforms all the other DE-variants in terms of final accuracy over two instances of the radar poly-phase code design problem as well as the FMS problem.

6.5 Selection of the Neighborhood Size

The proper selection of the neighborhood's size (equal to $2k+1$, where k is the neighborhood radius) in DEGL affects the trade-off between exploitation and exploration. For solving any given optimization problem, this selection remains an open problem. In practice, it is up to the practitioner and it is based solely on his/her experience. Some empirical guidelines may, however, be provided based on the fact that if the neighborhood size is large (near the population size), then because of the overlapping of the neighborhoods of successive vectors, neighborhood-best of a number of vectors can be similar to the globally best vector in the entire population. This again increases the attraction of most of the vectors towards a specific point in the search space and results in loss of the explorative power of the algorithm. Our experiments suggest that a neighborhood size that is above 40% of the population size makes the performance of DEGL comparable to that of the DE/target-to-best/1/bin. Again too small a neighborhood runs the risk of losing diversity of the population, as the difference vector in the local mutation model (equation (14)) may become too small. This is due to the fact that the vectors belonging to a small neighborhood may quickly become very similar to each other. We empirically observe that for $NP = 10 \cdot D$, the overall performance of the algorithm is not very sensitive to the neighborhood size varying between 10% and 20% of *NP*. Other choices for the population size *NP* and the corresponding radius of the neighborhood are topics of future research.

Below we provide the overall success rate of the DEGL/SAW algorithm for neighborhood size varying from 5% to 70% of *NP*, over 100-dimensional multi-modal functions f_{10} and f_{11}. Since both the functions have their optima at the origin (0), we plot the percentage of runs that successfully yielded a final accuracy below 10^{-15} for different neighborhood sizes. We relaxed the threshold objective function value from 10^{-20}, so that at least one run of DEGL for all neighborhood sizes may converge below the threshold value.

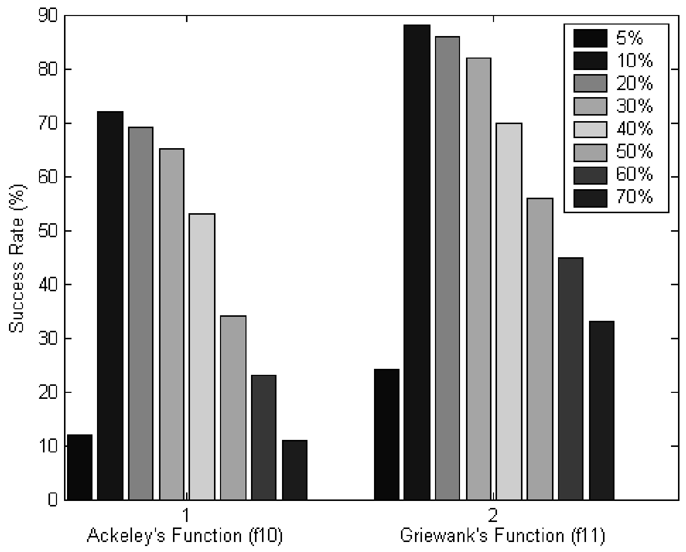


Fig. 12. Variation of the overall success rate of DEGL/SAW with increasing neighborhood size (for 100-dimensional functions f_{10} and f_{11}). Neighborhood sizes are indicated in the legend.

Thorough experimentation with all the test problems shows that a neighborhood size of around 10% provides reasonably accurate results with high success rates over most of the benchmark problems covered here.

6.6 Correlation Between the Neighborhood Size and Weight Factor

Both the neighborhood size and the weight factor w are related to the balancing of the explorative and exploitative tendencies of DEGL. Establishment of any theoretical correlation between these two parameters remains an interesting problem for future research. In this section we provide a discussion on such correlation, based on our empirical results on the benchmark functions.

If we keep w constant throughout, then for neighborhood sizes ($2k+1$, where k is the neighborhood radius) varying between approximately 15% to 25% of *NP*, reasonably good accuracy is achieved with $0.45 < w < 0.55$ over most of the uni- and multi-modal benchmarks. Larger values of w in [0.7, 1.0], result in marginally better results compared to DE/target-to-best/1/bin but comparable or worse than one or more DE-variants tested here. However, for still smaller neighborhood size varying between 5% to 15% of *NP*, the optimal range of w for best accuracy is observed in [0.6, 0.75]. For neighborhood sizes roughly above 65% of the population size *NP* none of the time-varying weight factor schemes (described in Section 4.4) provided significant improvement of DEGL over DE/target-to-best/1/bin. This is expected because when the neighborhood size approaches the population size, the global and local mutation models do not differ significantly with respect to their best vectors and the role of weight factor becomes less prominent.

In the case when w is made self-adaptive, if the neighborhood-size is below 30% of *NP*, DEGL exhibits an evolutionary learning strategy that initially promotes exploration of the feasible search volume, but during the later stages of search favors exploitation and thus aids quick convergence to the global optimum. This trend has also been shown in Figure 6 for various benchmark functions. However, we observe that if the neighborhood size is increased beyond 30%, the evolutionary learning gradually becomes erratic and for neighborhood sizes beyond 60% of *NP*, the self-adaptive characteristics of w become almost random over generations for most of the benchmarks. This tendency has been shown in Figure 13 for the generalized Ackley's function f_{10}. This figure indicates that if the neighborhood size approaches *NP*, the adaptation mechanisms of w can hardly guide the search . We intend to investigate these facts more thoroghly in a future communication.

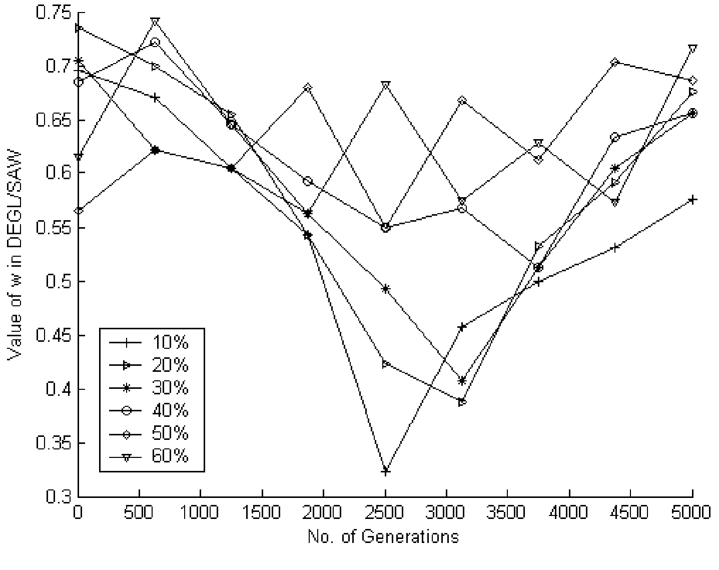


Fig.13. Self-adaptation characteristics of the best vector of the DEGL/SAW scheme on the generalized Ackley's function (f_{10}) for different neighborhood sizes.

7. Conclusions and Future Work

In this study we proposed a hybrid DE-type mutation/recombination operator that is a linear combination of two other mutation/recombination operators (an explorative and an exploitive operator), in an attempt to balance their effects. The new operator depends on a user-defined weight factor w. To circumvent the problem of determining a proper value of w for each problem, we proposed six different schemes for selecting and tuning this parameter. Among these, the self-adaptive weight scheme performed best on most of the benchmark functions tested.

The neighborhood-based DE mutation, equipped with self-adaptive weight factor, attempts to make a balanced use of the exploration and exploitation abilities of the search mechanism and is therefore more likely to avoid false or premature convergence in many cases. An extensive performance comparison with five significant DE variants and four other state-of-the-art evolutionary optimization techniques indicated that the proposed approaches enhance DE's ability to accurately locate solutions in the search space. The use of the self-adaptive mutation scheme can lead to reliable optimization since it alleviates the problems generated by poor trade-off between the explorative and exploitative tendencies of the algorithm, such as decreased rate of convergence, or even divergence and premature saturation.

This, however, does not lead us to claim that the DEGL family of algorithms may outperform their contestants over every possible objective function since it is impossible to model all possible complexities of real-life optimization problems with the limited test-suite that we used for testing the algorithms. In addition, the performance of the competitor DE variants may also be improved by blending other mutation strategies with judicious parameter tuning, a topic of future research. The conclusion we can draw at this point is that DE with the suggested modifications can serve as an attractive alternative for optimizing a wide variety of objective functions.

The present work can be extended in several directions. Future research may focus on providing some empirical or theoretical guidelines for selecting the neighborhood size over different types of optimization problems. The effect of other neighborhood topologies (star-shaped, wheel-shaped, fully connected, etc.) on the performance of DEGL should be investigated theoretically. It would be interesting to study the performance of the DEGL family when the various control parameters (*NP*, *F*, and *Cr*) are self-adapted following the ideas of the SADE algorithm [28].

Acknowledgements: We are grateful to the editor-in-chief, the anonymous associate editor and the anonymous reviewers for their very detailed comments and criticisms that helped a lot to improve the presentation of the paper.

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